COURSE: INVESTMENTS |CHAPTER 5

RISK AND RETURN

Dr Abenet Yohannes, Ph.D.

Chapter Purpose

Investment decisions involve two fundamental questions.

What return can I expect?

The reward side of the decision.

What risk must I accept?

The uncertainty surrounding that reward.

This chapter uses historical investment performance to develop the core concepts:

- Return · average return · risk · volatility
- Risk premium · inflation · real return
- Probability · Value at Risk · Expected Shortfall

Learning Objectives

By the end of this chapter, students should be able to:

Measure return

Holding-period return; income vs capital gain; arithmetic and geometric averages; expected return.

Measure risk

Variance and standard deviation; probability distributions; reward-to-volatility measures.

Interpret evidence

Historical risk–return relationships; risk premiums; nominal vs real returns and inflation.

Assess downside

Value at Risk and Expected Shortfall; using historical evidence appropriately.

What Is Investment Return?

Investment return measures the financial outcome from committing capital to an asset.

Income

Dividends
Interest

Price change

Capital gain
Capital loss

Total return

Income + price change

Both sources must be counted to describe performance fully.

Holding-Period Return

The holding-period return (HPR) measures the total return earned during the period an investment is held.

$$HPR = (P_1 - P_0 + D) / P_0$$

P_0

Beginning price

P_1

Ending price

D

Income received during the period

Holding-Period Return — Worked Example

An investor buys a share for ETB 100. One year later the share price is ETB 110 and a dividend of ETB 5 has been received.

$$HPR = (110 - 100 + 5) / 100$$
$$HPR = 15\%$$

Total return = 15%

Two Components of Return

Total return can be separated into two distinct components.

Income yield

Cash received relative to the amount invested.

Capital gains yield

Price appreciation relative to the amount invested.

```
HPR = Income yield + Capital gains yield
```

Income Yield

Income yield measures cash income relative to the initial investment.

```
Income yield = Income / Beginning price
```

Example: dividend = ETB 5, beginning price = ETB 100.

Income yield = 5%

Capital Gains Yield

$$\text{Capital gains yield} = (P_1 - P_0) / P_0$$

$$(110 - 100) / 100 = 10\%$$

Income yield

5%

Capital gains yield

10%

Total return

15%

Capital Loss

Purchase price = ETB 100 · ending price = ETB 90 · dividend = ETB 4.

$$HPR = (90 - 100 + 4) / 100$$

$$\mathbf{HPR = -6\%}$$

Income does not necessarily prevent an overall loss

Why Average Returns Matter

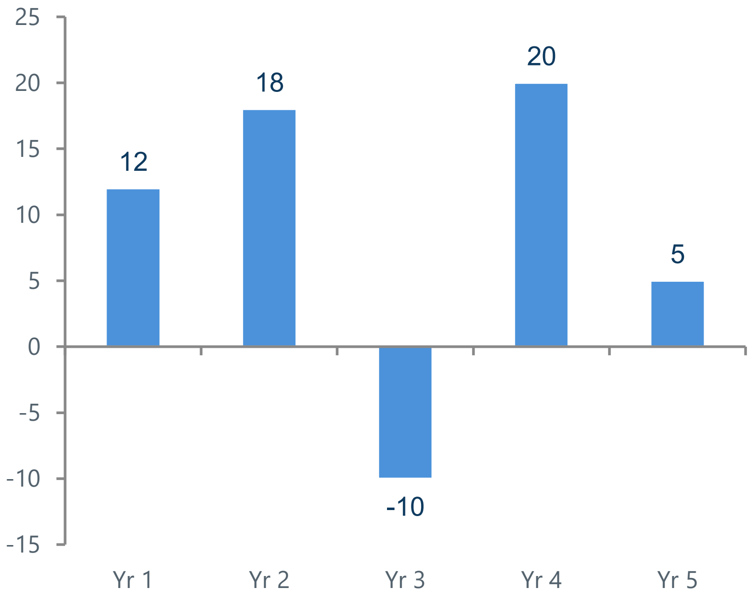
Investment returns change from year to year. How should a series of annual returns be summarized?

Arithmetic mean

Simple average of periodic returns.

Geometric mean

Compounded growth rate of wealth.



Five-year return series (%)

Arithmetic Average Return

The arithmetic average is the simple average of the periodic returns.

$$\bar{R} = (R_1 + R_2 + \cdots + R_n) / n$$

It represents the typical outcome of a single period, and is the natural starting point for forming expectations about the next period.

Arithmetic Average — Example

Returns: 10%, 20%, −5%, 15%.

$$\bar{R} = (10 + 20 - 5 + 15) / 4$$
$$\bar{R} = 10\%$$

Arithmetic average return = 10%

Geometric Average Return

The geometric average incorporates compounding.

$$R_G = \left[(1+R_1)(1+R_2)\cdots(1+R_n) \right]^{(1/n)} - 1$$

It measures the constant annual compounded rate that would produce the same ending wealth as the actual sequence of returns.

Why the Two Means Differ

Year 1

+50%

Year 2

−50%

```
Arithmetic average = (50% - 50%) / 2 = 0%
```

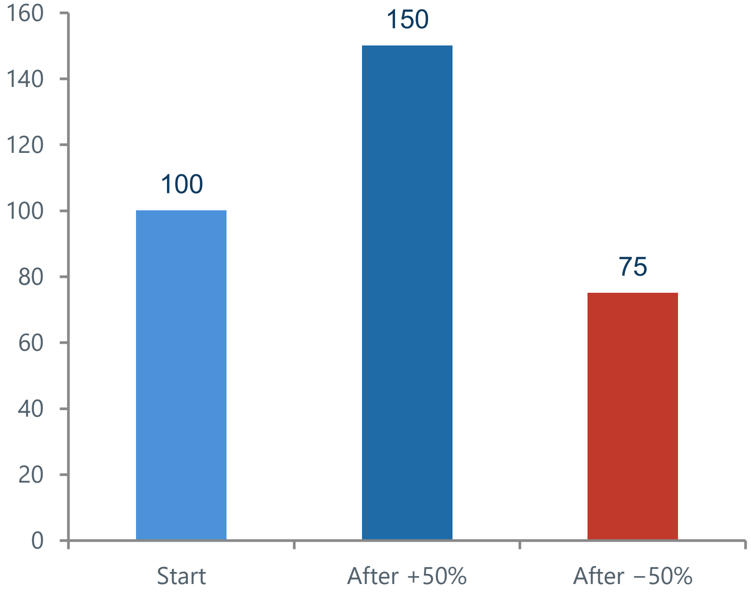
Does a 0% average mean the investor broke even?

No — the wealth outcome is a loss

The Volatility Effect

$$100 \times 1.50 = 150$$
$$150 \times 0.50 = 75$$

Ending wealth is ETB 75 despite an arithmetic average return of 0%. Variability itself erodes compounded wealth.



Wealth path: ETB 100 → 150 → 75

Arithmetic vs Geometric Mean

Arithmetic mean

Useful for describing the average one-period outcome and for forming forward-looking expectations.

Geometric mean

Useful for describing compounded historical growth and actual wealth accumulation.

```
Geometric mean ≤ Arithmetic mean   (when returns vary)
```

The gap between them widens as volatility increases.

Historical vs Expected Return

Historical return

What actually happened in the past — the evidence available for analysis.

Expected return

What investors anticipate for the future — the basis for decisions.

Investment decisions should ultimately depend on expectations about future outcomes — not simply on historical averages.

Expected Return

Expected return is the probability-weighted average of possible future returns.

$$E(R) = \Sigma\ p_s \cdot R_s$$

p_s

Probability of scenario s

R_s

Return under scenario s

Expected Return — Example

Scenario	Probability	Return
Boom	20%	30%
Normal	50%	12%
Recession	30%	−10%

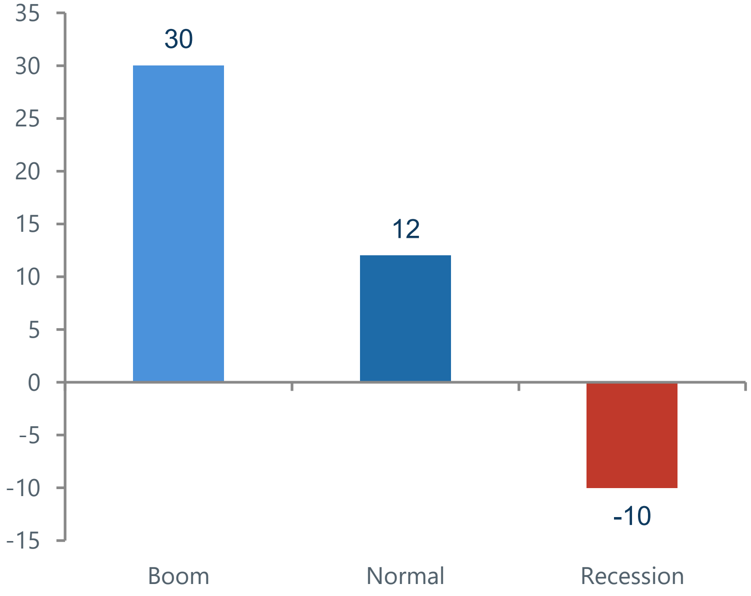
$$E(R) = 0.20(30\%) + 0.50(12\%) + 0.30(-10\%)$$
$$\mathbf{E(R) = 9\%}$$

Expected Return Is Not Guaranteed

An expected return of 9% does not mean the investment will earn exactly 9%.

The actual outcome could be 30%, 12%, −10%, or something else entirely if the assumed scenarios are incomplete.

Expected return represents an average expectation, not a promise



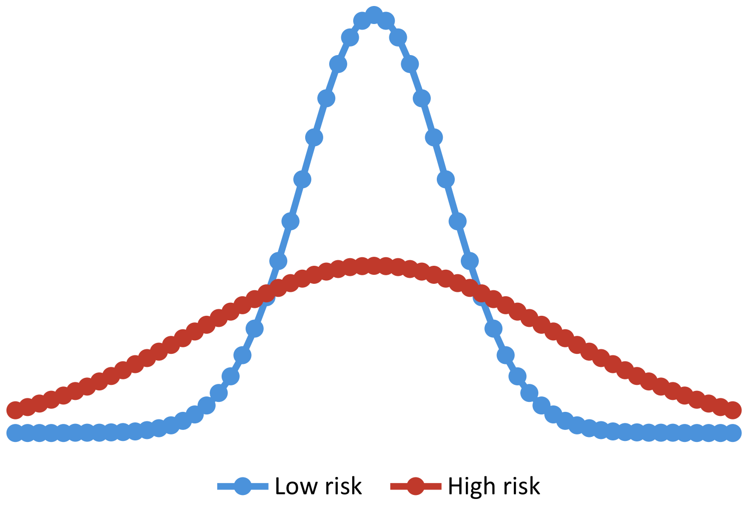
Possible outcomes around E(R) = 9%

What Is Investment Risk?

Investment risk is the uncertainty surrounding future investment outcomes.

More uncertainty = greater risk

An investment with widely dispersed possible returns is generally considered riskier than one with tightly clustered returns.



Narrow vs wide return distributions

Risk Is More Than Losing Money

Risk takes several forms beyond an outright loss of capital:

Returns below expectations

Outcome falls short of what was required.

Capital loss

The invested principal itself declines.

Income uncertainty

Dividends or interest may be reduced.

Purchasing-power loss

Inflation erodes real value.

Default

The counterparty fails to pay.

Liquidity and extreme events

Exit is costly; tail outcomes occur.

Major Sources of Investment Risk

Market risk
Broad price movements

Business risk
Firm-level performance

Credit risk
Borrower default

Interest-rate risk
Rate-driven revaluation

Inflation risk
Purchasing-power erosion

Liquidity risk
Costly or delayed exit

Currency risk
Exchange-rate movements

Political / regulatory
Policy and rule changes

Probability Distribution

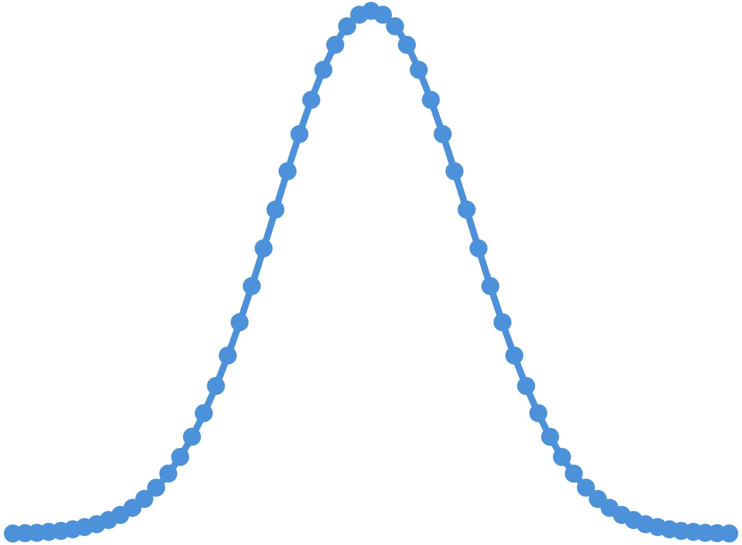
A probability distribution shows the possible investment outcomes and the probability attached to each one.

Expected return

The centre of the distribution.

Risk

The spread around that centre.



Bell-shaped distribution of returns

Narrow Distribution

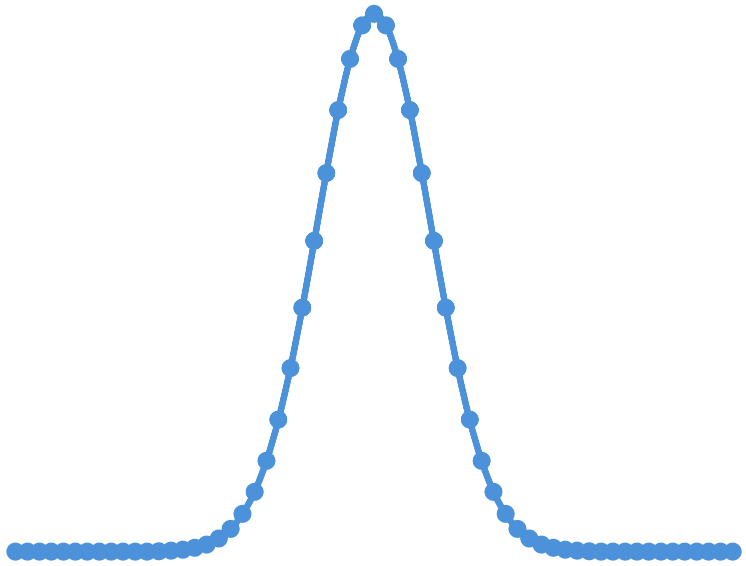
A narrow return distribution indicates outcomes clustered close to the expected return.

Lower variability

Outcomes are more predictable.

Lower risk

Less uncertainty to bear.



Tightly clustered outcomes

Wide Distribution

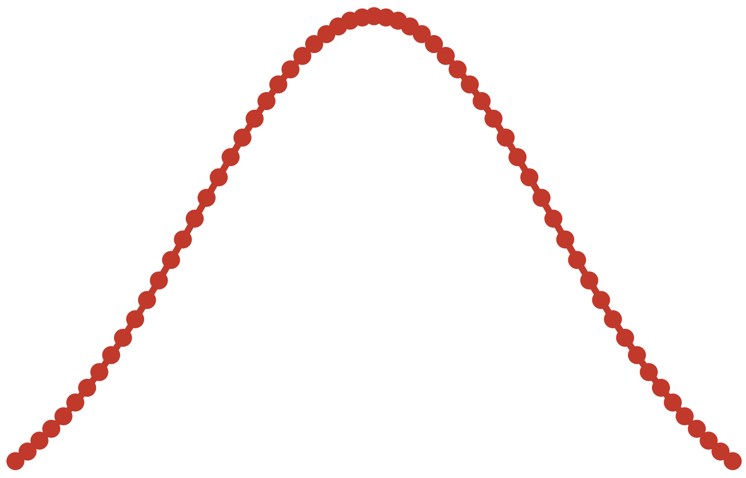
A wide return distribution indicates outcomes spread farther from the expected return.

Greater variability

Outcomes are less predictable.

Greater risk

More uncertainty to bear.



Widely dispersed outcomes

Measuring Risk

Two statistical measures summarize how far returns tend to fall from their expected or average value.

Variance

The average squared deviation from the mean.

Standard deviation

The square root of variance, expressed in return units.

Both describe the dispersion of returns — the working definition of risk in this chapter.

Variance

For a probability distribution of returns:

$$\sigma^2 = \Sigma\ p_s \cdot [\ R_s - E(R)\ ]^2$$

Because deviations are squared, variance gives greater weight to outcomes that fall farther from the expected return — which is precisely why it captures the intuition of risk.

Standard Deviation

Standard deviation is the square root of variance, and is measured in the same units as returns.

$$\sigma = \sqrt{\sigma^2}$$

Higher σ

Greater volatility

Lower σ

Lower volatility

Standard Deviation

Imagine two buses.

Bus A

Usually arrives within 1 minute of schedule.

Bus B

Sometimes 20 minutes early, sometimes 30 minutes late.

Even if both have the same average arrival time, Bus B is far less predictable.

That unpredictability is higher standard deviation

Comparing Two Investments

Investment A

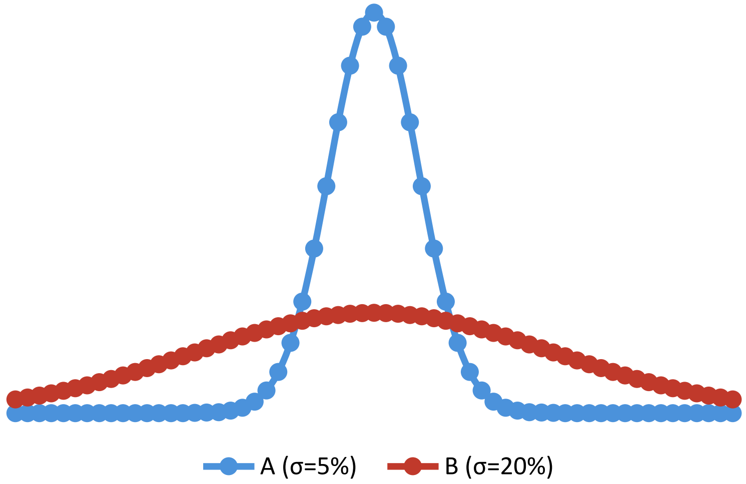
E(R) = 10%
σ = 5%

Investment B

E(R) = 10%
σ = 20%

Both offer the same expected return.

Investment B is considerably more volatile



Same centre, different width

Why Study Capital-Market History?

Historical data help investors understand what markets have actually delivered:

Typical returns

Long-run averages by asset class

Volatility

How widely outcomes varied

Market cycles

Expansions, drawdowns, recoveries

Losses

Frequency and depth of declines

Risk premiums

Reward above the risk-free rate

Relationships

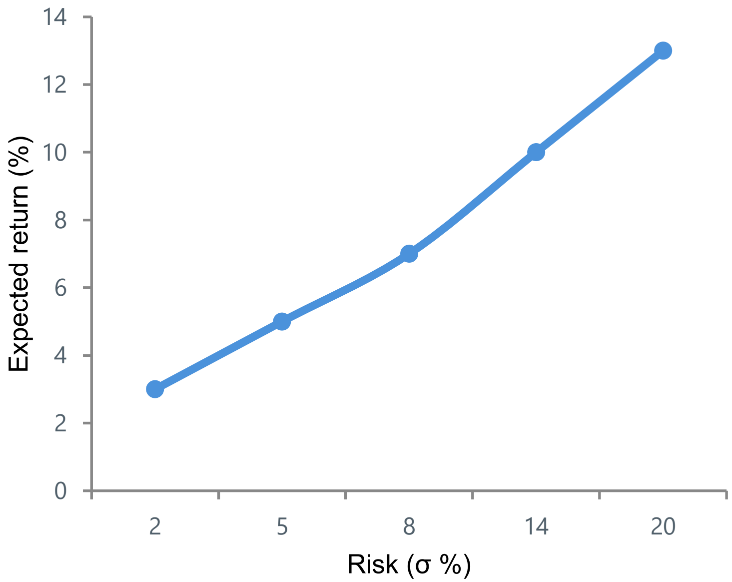
How asset classes move together

The Broad Historical Pattern

Over long periods, investment markets have generally demonstrated a relationship between risk and expected reward.

Assets carrying greater uncertainty have typically had to offer higher expected returns to attract capital.

> ***Greater uncertainty has historically required greater expected compensation.***

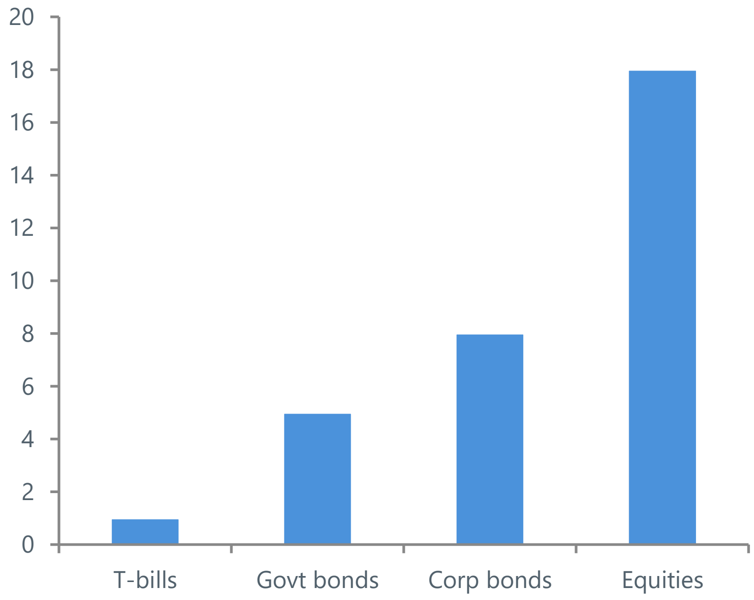


Expected return rises with risk

Illustrative Risk Hierarchy

1. Treasury bills — lower volatility
2. Government bonds
3. Corporate bonds
4. Diversified equities — higher volatility

The exact realized ranking and returns vary by period and by market.



Illustrative volatility by asset class

The Risk and Return Principle

Higher expected return

generally requires…

Greater risk bearing

…accepting more uncertainty

Higher risk does not guarantee higher realized return.

Risk means outcomes are uncertain. A higher expected return is compensation for that uncertainty — not a payment that is certain to arrive.

What Is a Risk Premium?

A risk premium is the additional expected return investors require for bearing risk above a risk-free benchmark.

$$\text{Risk premium} = \text{Expected return} - \text{Risk-free rate}$$

It is the price of risk: the extra reward that persuades an investor to give up certainty.

Risk Premium — Example

Expected stock return

12%

Risk-free rate

4%

Risk premium = 12% - 4% = 8%

Risk premium = 8%

Why Does a Risk Premium Exist?

Suppose investors can earn 4% with minimal assumed risk. Why would they choose a volatile investment also expected to earn 4%?

They would require additional expected compensation

The premium exists because most investors are risk-averse: they must be paid to accept uncertainty they could otherwise avoid.

The Equity Risk Premium

The equity risk premium is the expected additional return from equities relative to an appropriate risk-free benchmark.

```
Equity risk premium = Expected equity return − Risk-free rate
```

Asset pricing

Inputs to required-return models

Valuation

Discount rates for cash flows

Portfolio management

Allocation between risky and safe assets

Inflation Changes Investment Outcomes

Investment grows by

10%

Prices increase by

7%

Has purchasing power increased by 10%? No — the price of what the money buys has risen too.

Nominal return

Measured in money terms

Real return

Measured in purchasing power

Nominal Return

Nominal return is the percentage return measured in current monetary units. It does not adjust for inflation.

ETB 100 → ETB 110

Nominal return = 10%

Real Return

Real return measures growth in purchasing power after accounting for inflation.

$$1 + r_{real} = (1 + r_{nominal}) / (1 + i)$$
$$r_{real} = (1 + r_{nominal}) / (1 + i) - 1$$

where i is the rate of inflation over the same period.

Real Return — Example

Nominal return

10%

Inflation

7%

```
r_real = 1.10 / 1.07 - 1
r_real ≈ 2.80%
```

Real return ≈ 2.8%

A Useful Approximation

When rates are relatively modest, real return can be approximated by subtraction.

$$\text{Real return} \approx \text{Nominal return} - \text{Inflation}$$

$$\mathbf{10\% - 7\% = 3\%}$$

Approximation

3.0%

Exact result

2.8%

Why Real Return Matters

What can my future wealth actually buy?

An investment may produce positive nominal returns while generating low or even negative real returns if inflation is sufficiently high.

Purchasing power — not money value — is the true measure of success

Return Alone Is Not Enough

Fund A

Return = 15%
Risk = 10%

Fund B

Return = 17%
Risk = 30%

Can we conclude that Fund B is better? No — it earns 2 percentage points more while carrying three times the volatility.

Judge return relative to risk

The Reward-to-Volatility Concept

One common framework compares excess return with volatility. The most widely used measure is the Sharpe ratio.

$$\text{Sharpe ratio} = [\, E(R) - R_f \,] / \sigma$$

Higher values indicate more excess return earned per unit of total volatility — a better bargain for the risk taken.

Sharpe Ratio — Example

Portfolio return

14%

Risk-free rate

4%

Standard deviation

20%

```
Sharpe = (14% − 4%) / 20%
Sharpe = 0.50
```

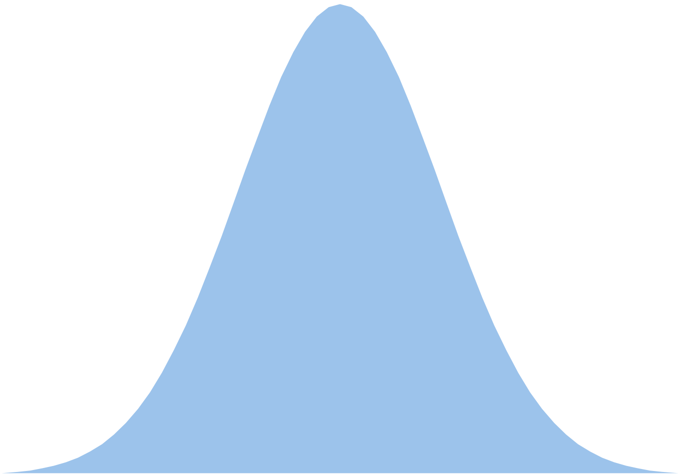
0.50 units of excess return per unit of volatility

Is Standard Deviation Enough?

Standard deviation treats positive surprises and negative surprises identically — both are simply deviations from the mean.

But investors are especially concerned about large losses.

This asymmetry of concern motivates dedicated downside-risk measures.



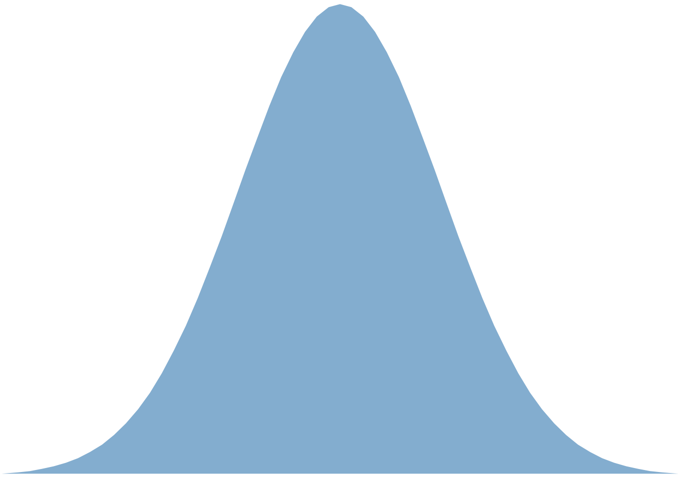
Symmetric measure, asymmetric concern

Value at Risk (VaR)

Value at Risk estimates a loss threshold over a specified horizon at a stated confidence level.

> ***There is a 5% probability that the portfolio will lose more than ETB 1 million over the specified period.***

VaR converts a distribution into a single, communicable statement about how bad things could plausibly get.



The highlighted loss tail

VaR Requires Three Elements

1. Loss amount — e.g. ETB 1 million
2. Time horizon — e.g. one day
3. Confidence level — e.g. 95%

ETB 1 million	One day	95%
Loss amount	Horizon	Confidence

Without all three elements, a VaR figure is incomplete and cannot be interpreted.

The Limitation of VaR

VaR tells us

Approximately where the loss threshold begins.

VaR does not tell us

How severe losses may become once that threshold is exceeded.

For that, we use Expected Shortfall

Expected Shortfall

Expected Shortfall estimates the average loss conditional on being in the worst tail beyond the VaR threshold.

> ***If the extreme-loss region occurs, how bad is the loss on average?***

It is also referred to as Conditional VaR in some contexts, and is increasingly used in risk regulation and portfolio oversight.

VaR vs Expected Shortfall

Value at Risk

How bad can the loss threshold be at a specified confidence level?

Expected Shortfall

If we enter the extreme-loss tail, what is the average loss?

Expected Shortfall adds information about tail risk

Compounding Matters

Investment

ETB 100,000

Return

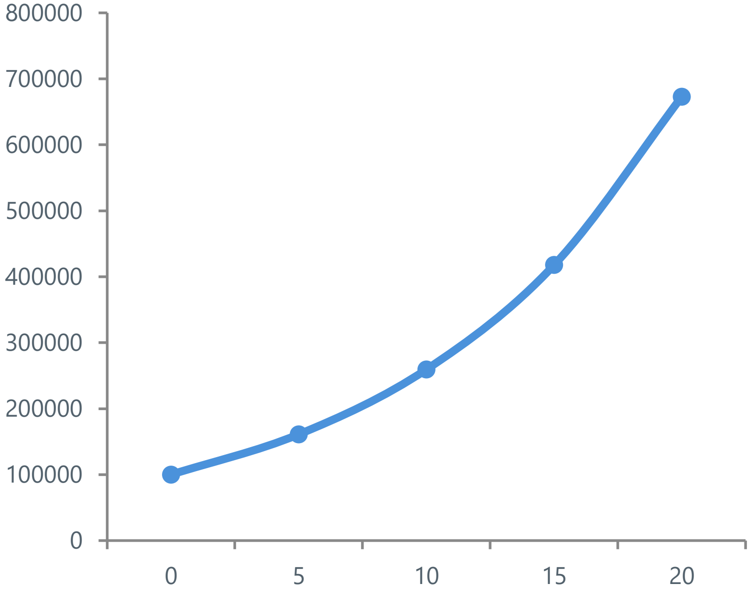
10% p.a.

Period

20 years

$$FV = 100{,}000 \times (1.10)^{20} \approx \text{ETB } 672{,}750$$

Before taxes, fees and inflation.



Growth of ETB 100,000 at 10% p.a.

Time Magnifies Small Differences

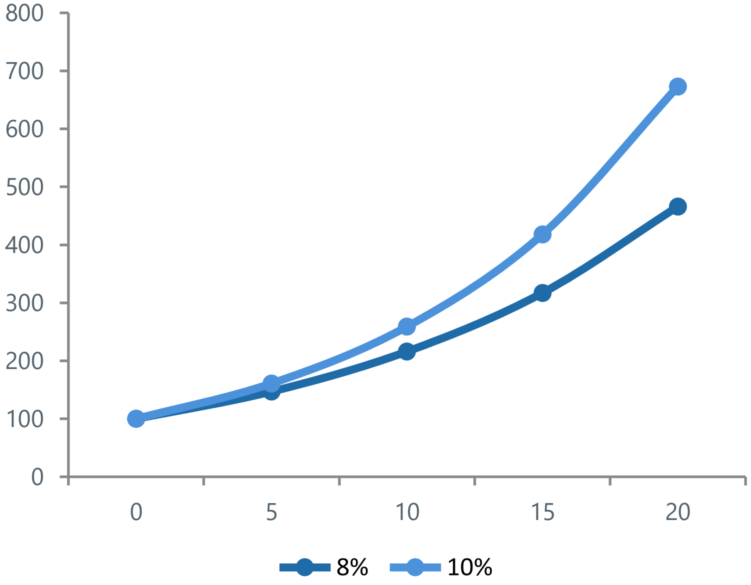
Portfolio A

8% annual return

Portfolio B

10% annual return

Over one year the difference is just 2 percentage points. Over decades, the difference in accumulated wealth becomes substantial because of compounding.



Two wealth curves over 20 years

Volatility Also Affects Compounding

Year 1

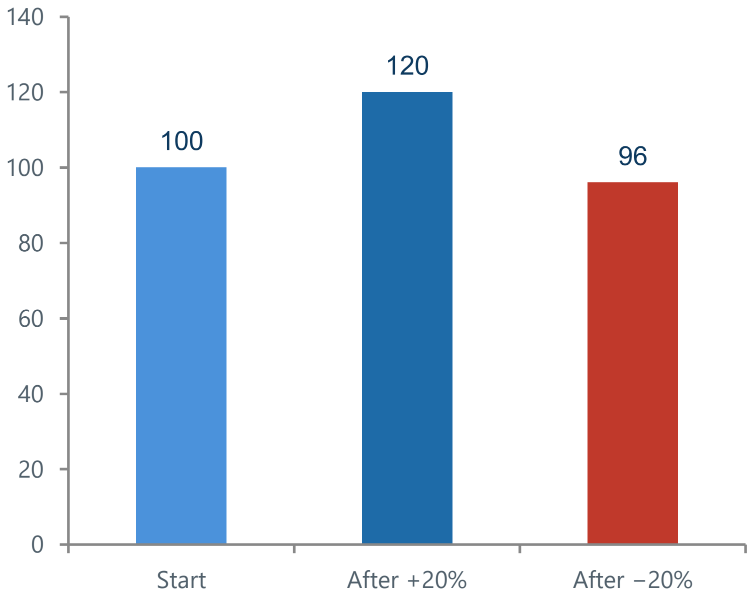
+20%

Year 2

−20%

```
Arithmetic average = 0%
1.20 × 0.80 = 0.96  →  wealth falls 4%
```

This is known as volatility drag



100 → 120 → 96

What Can History Teach Us?

Historical data provide important evidence for investment analysis. They help estimate:

Average returns
Long-run central tendency

Volatility
Typical dispersion of outcomes

Correlations
How assets move together

Risk premiums
Reward above the risk-free rate

Frequency of losses
How often declines occurred

Market cycles
Patterns of expansion and stress

But History Is Not the Future

Future markets may differ from the past because of:

- Economic change · regulation · technology
- Demographics · inflation · interest rates
- Political conditions · market structure

Historical return is evidence — not a promise.

Common Mistakes With Historical Returns

Assuming repetition

Expecting past returns to recur

Ignoring inflation

Confusing nominal with real

Ignoring risk

Ranking on return alone

Ignoring fees

Overstating net outcomes

Short periods

Too little data to be meaningful

Ignoring extremes

Overlooking tail events

Wrong comparisons

Mismatched asset classes

Mean confusion

Arithmetic used as geometric

Practical Exercise — Risk and Return Analysis

Measure	Investment A	Investment B
Expected return	10%	15%
Standard deviation	8%	22%
Risk-free rate	4%	4%
Inflation	6%	6%

Student tasks

1. Calculate the risk premium of each investment.
2. Approximate each real expected return.
3. Calculate the Sharpe ratio for each.
4. Which investment has greater total volatility?
5. Which provides more expected excess return per unit of volatility?
6. Would the answer be the same for every investor?
7. What additional information would you request before investing?

Key Takeaways

Measuring return

Return is income plus capital gain or loss. HPR captures total performance; arithmetic describes the average period, geometric the compounded growth; expected return is probability-weighted.

Measuring risk

Risk is uncertainty about outcomes. Variance and standard deviation measure dispersion; risk-adjusted measures compare reward with risk; VaR and Expected Shortfall describe the loss tail.

Interpreting evidence

History links risk to expected reward and pays a risk premium above the risk-free rate. Real return, not nominal, measures purchasing power — and compounding makes both return and volatility decisive over long horizons.

Return tells us the reward. Risk tells us the uncertainty surrounding that reward. Good investment decisions require understanding both.

Next Chapter — Efficient Diversification

Chapter 6 turns from single investments to portfolios: how combining assets changes the risk an investor actually bears.

Portfolio mathematics

Expected portfolio return, variance and standard deviation, covariance and correlation.

Diversification

Systematic vs firm-specific risk, two-asset portfolios, portfolio opportunity sets.

Optimal choice

Efficient frontier, minimum-variance and efficient portfolios, the risk-free asset and the Capital Allocation Line, the optimal risky portfolio, and risk aversion.

THANK YOU

Questions and discussion

Dr Abenet Yohannes, Ph.D.

abenetyohannes@gmail.com

abenetyohannes.com