02

Time Value of Money

Presented by

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Learning Objectives

By the end of this chapter, participants will be able to:

- Explain the meaning of the time value of money
- Distinguish present value from future value
- Explain simple and compound interest
- Calculate future and present values of single amounts
- Apply annual and multiple-period compounding
- Calculate effective annual rates
- Value ordinary annuities and annuities due
- Calculate perpetuity values
- Value uneven cash-flow streams
- Apply discounting to investment decisions
- Prepare loan-amortization schedules
- Use financial calculators and spreadsheet functions
- Apply TVM concepts to savings, loans, bonds and capital budgeting

Chapter Roadmap

This chapter covers:

- Meaning of the time value of money
- Cash-flow timelines
- Simple interest
- Compound interest
- Future value
- Present value
- Compounding frequency
- Effective annual rate
- Annuities
- Perpetuities
- Uneven cash flows
- Loan amortization
- Inflation and real value
- Financial applications

Section 1

INTRODUCTION

Introduction to the Time Value of Money

What Is the Time Value of Money?

The time value of money means that:

> *A unit of money received today is worth more than the same unit received in the future.*

Money available today can:

- Be invested
- Earn interest
- Generate income
- Reduce debt
- Provide liquidity
- Protect against uncertainty

Why Money Has Time Value

Money has time value because of:

- Earning capacity
- Inflation
- Risk
- Liquidity preference
- Opportunity cost
- Uncertainty about future payment

Earning Capacity

Money received today can be invested to earn a return.

Example:

- Principal: ETB 10,000
- Annual return: 10%
- Horizon: 1 year

$$\text{ETB } 10{,}000 \times (1 + 0.10) = \text{ETB } 11{,}000$$

Therefore, ETB 10,000 today is worth more than ETB 10,000 received one year later.

Inflation

Inflation reduces purchasing power over time.

Example:

- Annual inflation: 8%
- Item cost today: ETB 10,000

$$10{,}000 \times (1.08) = \text{ETB } 10{,}800$$

An item costing ETB 10,000 today may cost approximately ETB 10,800 next year.

Risk and Uncertainty

Future payments may not be received as expected because of:

- Default
- Business failure
- Political instability
- Currency changes
- Market uncertainty
- Delayed payment
- Unexpected events

Investors require compensation for accepting uncertainty.

Liquidity Preference

People generally prefer money today because it provides:

- Immediate purchasing power
- Flexibility
- Emergency protection
- Investment opportunities
- Reduced financial uncertainty

Opportunity Cost

Opportunity cost is the return sacrificed by choosing one use of money instead of another.

Example:

- Available funds: ETB 100,000
- Alternative return: 12%
- Chosen project return: 8%

Using the funds in a project that earns only 8% creates an opportunity cost relative to the 12% alternative.

Applications of TVM

The time value of money is used in:

- Capital budgeting
- Stock valuation
- Bond valuation
- Loan decisions
- Mortgage calculations
- Retirement planning
- Savings decisions
- Lease analysis
- Business valuation
- Investment appraisal

Section 2

TIMELINES

Cash-Flow Timelines

What Is a Cash-Flow Timeline?

A cash-flow timeline shows:

- Timing of cash flows
- Amounts received
- Amounts paid
- Interest rate
- Number of periods

It helps organize financial calculations.

Basic Timeline

- Time 0 represents today
- Time 1 represents the end of Period 1
- Time 2 represents the end of Period 2
- Time 3 represents the end of Period 3

Cash Inflows and Outflows

Cash Inflow

Money received.

- Revenue
- Investment return
- Loan receipt
- Asset sale

Cash Outflow

Money paid.

- Investment cost
- Loan repayment
- Operating expense
- Asset purchase

Financial Symbols

Symbol	Meaning
PV	Present value
FV	Future value
r	Interest or discount rate
n	Number of periods
PMT	Periodic payment
CF_t	Cash flow at time t

Important TVM Rule

> *Cash flows can only be compared or added when they are expressed at the same point in time.*

Therefore:

- Move future cash flows to the present through discounting
- Move present cash flows to the future through compounding

Section 3

INTEREST

Interest Concepts

What Is Interest?

Interest is compensation paid for the use of money.

Lender's Perspective

- Income
- Return
- Compensation for risk

Borrower's Perspective

- Financing cost
- Cost of using borrowed funds

Principal

Principal is the original amount:

- Borrowed
- Lent
- Invested
- Deposited

Interest is calculated using the principal and the applicable rate.

Interest Rate

The interest rate is normally expressed as a percentage of principal per period.

Examples:

- 10% per year
- 2% per month
- 5% per quarter
- 12% nominal annual rate compounded monthly

Simple vs Compound Interest

Simple Interest

Interest is calculated only on the original principal.

Compound Interest

Interest is calculated on:

- Original principal
- Previously earned interest

Compound interest produces interest on interest.

Section 4

SIMPLE INTEREST

Simple Interest

Simple-Interest Formula

$$I = P \times r \times n$$

Where:

- I = interest
- P = principal
- r = interest rate per period
- n = number of periods

Simple Future Value

$$FV = P(1 + rn)$$

This formula adds simple interest to the original principal.

Simple-Interest Example

Assume:

- Principal: ETB 100,000
- Annual interest: 10%
- Time: 3 years

$$I = 100{,}000 \times 0.10 \times 3$$

$$I = \text{ETB } 30{,}000$$

Simple Future Value Example

$$FV = 100{,}000\,(1 + 0.10 \times 3)$$

$$FV = ETB\ 130{,}000$$

Limitation of Simple Interest

Simple interest does not recognize interest earned on previous interest.

It is commonly used for:

- Short-term loans
- Trade credit
- Treasury instruments
- Certain informal agreements
- Short-duration investments

Section 5

COMPOUND INTEREST

Compound Interest

Compound Interest

> *Interest earned in one period becomes part of the principal for the next period.*

The investment grows at an increasing rate.

Compound Future-Value Formula

$$FV_n = PV(1 + r)^n$$

Where:

- FV_n = value after n periods
- PV = initial investment
- r = periodic interest rate
- n = number of periods

Compound Future-Value Example

Assume:

- Present value: ETB 100,000
- Annual return: 10%
- Investment period: 3 years

$$FV = 100{,}000\ (1.10)^3$$

$$FV = ETB\ 133{,}100$$

Year-by-Year Growth

Year	Beginning Balance	Interest at 10%	Ending Balance
1	100,000	10,000	110,000
2	110,000	11,000	121,000
3	121,000	12,100	133,100

Simple vs Compound Interest

Method	Value after Three Years
Simple interest	ETB 130,000
Compound interest	ETB 133,100
Difference	ETB 3,100

The difference becomes larger as time and interest rates increase.

Compound-Interest Effect

Compound growth is affected by:

- Initial amount
- Interest rate
- Time
- Compounding frequency

Higher rates and longer periods produce greater growth.

Rule of 72

The Rule of 72 estimates how long it takes an investment to double.

$$\text{Years to Double} \approx 72 \,/\, \text{Interest Rate}$$

Rule-of-72 Example

At an interest rate of 12%:

$$72 / 12 = 6$$

≈ 6 years to double

An investment will approximately double in six years. The rule provides an estimate, not an exact result.

Section 6

FUTURE VALUE

Future Value of a Single Amount

Meaning of Future Value

Future value is the amount that a current investment will grow to after earning interest for a specified period.

> *How much will today's money be worth in the future?*

Future-Value Formula

$$FV = PV(1 + r)^n$$

The term $(1 + r)^n$ is the future-value factor.

Future-Value Example

Assume:

- Investment: ETB 50,000
- Rate: 8%
- Period: 5 years

$$FV = 50{,}000\ (1.08)^5$$

$$FV \approx \text{ETB } 73{,}466$$

Effect of Interest Rate

Assume ETB 50,000 is invested for five years.

Interest Rate	Future Value
5%	ETB 63,814
8%	ETB 73,466
12%	ETB 88,117

Higher interest rates produce larger future values.

Effect of Time

Assume ETB 50,000 earns 8%.

Period	Future Value
1 year	ETB 54,000
3 years	ETB 62,986
5 years	ETB 73,466
10 years	ETB 107,946

Solving for the Interest Rate

When PV, FV and n are known:

$$r = (FV / PV)^{1/n} - 1$$

Interest-Rate Example

An investment grows from ETB 100,000 to ETB 161,051 in five years.

$$r = (161{,}051 \,/\, 100{,}000)^{1/5} - 1$$

$$r = 10\%$$

Solving for Time

When PV, FV and r are known:

$$n = \ln(FV / PV) / \ln(1 + r)$$

Time Example

How long will ETB 100,000 take to double at 12%?

$$n = \ln(200{,}000 / 100{,}000) / \ln(1.12)$$

$$n \approx 6.12 \text{ years}$$

Section 7

PRESENT VALUE

Present Value of a Single Amount

Meaning of Present Value

Present value is today's equivalent of a future amount.

> *How much is a future cash flow worth today?*

Present-Value Formula

$$PV = FV / (1 + r)^n$$

The term $1 / (1 + r)^n$ is the discount factor.

Present-Value Example

Assume:

- Future receipt: ETB 100,000
- Required return: 10%
- Time: 3 years

$$PV = 100{,}000 / (1.10)^3$$

$$PV \approx \text{ETB } 75{,}131$$

Interpretation

Receiving ETB 100,000 after three years is financially equivalent to receiving approximately ETB 75,131 today when the required return is 10%.

Effect of Discount Rate

Present value decreases when the discount rate increases.

Discount Rate	PV of ETB 100,000 in Five Years
5%	ETB 78,353
10%	ETB 62,092
15%	ETB 49,718

Effect of Time

Future value ETB 100,000, discount rate 10%. The further away a cash flow, the lower its present value.

Time	Present Value
1 year	ETB 90,909
3 years	ETB 75,131
5 years	ETB 62,092
10 years	ETB 38,554

Discounting and Compounding

Compounding

PV → FV

Moves money forward in time.

Discounting

FV → PV

Moves money backward in time.

They are inverse processes.

Section 8

COMPOUNDING

Compounding Frequency

Multiple Compounding Periods

Interest may be compounded:

- Annually
- Semiannually
- Quarterly
- Monthly
- Weekly
- Daily
- Continuously

More frequent compounding produces a higher future value.

General Compounding Formula

$$FV = PV(1 + r/m)^{mn}$$

Where:

- r = nominal annual rate
- m = compounding periods per year
- n = number of years

Semiannual Compounding

Assume:

- Investment: ETB 100,000
- Nominal rate: 12%
- Period: 3 years
- Compounding: semiannual

$$\text{Periodic rate} = 12\% / 2 = 6\%$$
$$\text{Number of periods} = 3 \times 2 = 6$$

Semiannual Future Value

$$FV = 100{,}000\ (1.06)^{6}$$

$$FV \approx \text{ETB } 141{,}852$$

Monthly Compounding

Assume:

- Investment: ETB 100,000
- Nominal annual rate: 12%
- Time: 3 years
- Monthly compounding

$$FV = 100{,}000\,(1 + 0.12/12)^{36}$$

$$FV \approx \text{ETB } 143{,}077$$

Compounding Comparison

ETB 100,000 invested for three years at a nominal annual rate of 12%:

Compounding	Future Value
Annual	ETB 140,493
Semiannual	ETB 141,852
Quarterly	ETB 142,576
Monthly	ETB 143,077

Present Value with Multiple Compounding

$$PV = FV / (1 + r/m)^{mn}$$

Where:

- Match the rate per period
- Match the number of periods

Section 9

EFFECTIVE RATES

Nominal and Effective Interest Rates

Nominal Annual Rate

A nominal annual rate is the stated annual rate before considering compounding within the year.

Example: 12% compounded monthly means:

- Nominal annual rate: 12%
- Monthly periodic rate: 1%

Effective Annual Rate

The Effective Annual Rate measures the actual annual return or financing cost after considering compounding.

$$EAR = (1 + r/m)^{m} - 1$$

EAR Example

Assume a nominal rate of 12% compounded monthly.

$$EAR = (1 + 0.12/12)^{12} - 1$$

$$EAR \approx 12.68\%$$

Why EAR Matters

EAR helps compare financial alternatives with different:

- Stated rates
- Compounding frequencies
- Loan structures
- Savings arrangements
- Investment products

The alternative with the lower nominal rate may not have the lower effective cost.

Rate Comparison Example

Investment	Nominal Rate	Compounding	Effective Rate
A	12.0%	Annual	12.00%
B	11.8%	Monthly	12.46%
C	12.0%	Quarterly	12.55%

Periodic Rate

```
Periodic Rate = Nominal Annual Rate / Periods per Year
```

All calculations must use the rate applicable to each cash-flow period.

Section 10

CONTINUOUS

Continuous Compounding

Continuous Compounding

Continuous compounding assumes interest is compounded at every possible instant.

$$FV = PV \cdot e^{rn}$$

Where $e \approx 2.71828$

Continuous-Compounding Example

Assume:

- Present value: ETB 100,000
- Annual rate: 10%
- Time: 3 years

$$FV = 100{,}000 \cdot e^{0.10 \times 3}$$

$$FV \approx \text{ETB } 134{,}986$$

Continuous Present Value

$$PV = FV \cdot e^{-rn}$$

Where:

- Advanced financial modeling
- Derivative pricing
- Economic analysis
- Certain investment models

Section 11

ANNUITIES

Annuities

What Is an Annuity?

An annuity is a series of equal cash flows occurring at regular intervals.

Examples include:

- Loan repayments
- Rent
- Pension payments
- Insurance premiums
- Regular savings
- Lease payments

Types of Annuities

Ordinary Annuity

Payments occur at the end of each period.

Annuity Due

Payments occur at the beginning of each period.

Ordinary Annuity Timeline

For a three-year ordinary annuity:

- Payment 1 occurs at the end of Year 1
- Payment 2 occurs at the end of Year 2
- Payment 3 occurs at the end of Year 3

Annuity-Due Timeline

For a three-year annuity due:

- Payment 1 occurs immediately
- Payment 2 occurs at the beginning of Year 2
- Payment 3 occurs at the beginning of Year 3

Section 12

FV — ORDINARY ANNUITY

Future Value of an Ordinary Annuity

Future-Value Formula

$$FV_{OA} = PMT \times \left[\left((1 + r)^n - 1\right) / r \right]$$

Where:

- PMT = periodic payment
- r = periodic rate
- n = number of payments

Ordinary-Annuity Example

Assume:

- Annual deposit: ETB 20,000
- Return: 10%
- Period: 5 years
- Deposits occur at year-end

$$FV = 20{,}000 \times \left[\left((1.10)^5 - 1\right) / 0.10 \right]$$

$$FV \approx \text{ETB } 122{,}102$$

Annuity Accumulation

Year-End Deposit	Time Earning Interest
Deposit 1	4 years
Deposit 2	3 years
Deposit 3	2 years
Deposit 4	1 year
Deposit 5	0 years

Section 13

FV — ANNUITY DUE

Future Value of an Annuity Due

Annuity-Due Future Value

$$FV_{AD} = FV_{OA} \times (1 + r)$$

Each payment earns interest for one additional period.

Annuity-Due Example

Using the earlier ordinary-annuity value:

$$FV_{AD} = 122{,}102 \times (1.10)$$

$$FV_{AD} \approx \text{ETB } 134{,}312$$

Ordinary Annuity vs Annuity Due

Feature	Ordinary Annuity	Annuity Due
Payment timing	End of period	Beginning of period
Interest periods	Fewer	One additional period
Future value	Lower	Higher
Present value	Lower	Higher

Section 14

PV — ORDINARY ANNUITY

Present Value of an Ordinary Annuity

Present-Value Formula

$$PV_{OA} = PMT \times [\ (1 - (1 + r)^{-n})\ /\ r\ ]$$

Ordinary-Annuity Present-Value Example

Assume:

- Annual payment: ETB 30,000
- Required return: 10%
- Payments: 4 years
- Payments occur at year-end

$$PV = 30{,}000 \times [\ (1 - (1.10)^{-4})\ /\ 0.10\ ]$$

$$PV \approx \text{ETB } 95{,}096$$

Interpretation

Receiving ETB 30,000 at the end of each year for four years is worth approximately ETB 95,096 today when the required return is 10%.

Section 15

PV — ANNUITY DUE

Present Value of an Annuity Due

Annuity-Due Present Value

$$PV_{AD} = PV_{OA} \times (1 + r)$$

Annuity-Due Example

$$PV_{AD} = 95{,}096 \times (1.10)$$

$$PV_{AD} \approx \text{ETB } 104{,}606$$

The value is higher because each payment is received one period earlier.

Section 16

ANNUITY PAYMENTS

Solving for Annuity Payments

Required Savings Payment

To determine the regular deposit needed to reach a future goal:

$$PMT = FV / [((1 + r)^n - 1) / r]$$

Savings Example

An investor wants ETB 1,000,000 after 10 years. Expected annual return: 10%.

$$PMT = 1{,}000{,}000 \;/\; [\; ((1.10)^{10} - 1) \;/\; 0.10 \;]$$

$$PMT \approx ETB\ 62{,}745$$

The investor should deposit approximately ETB 62,745 at each year-end.

Loan-Payment Formula

For a fully amortizing loan:

$$PMT = (PV \times r) / (1 - (1 + r)^{-n})$$

Loan-Payment Example

Assume:

- Loan: ETB 500,000
- Annual rate: 12%
- Term: 5 years
- Annual payments

$$PMT = (500{,}000 \times 0.12) / (1 - (1.12)^{-5})$$

$$PMT \approx ETB\ 138{,}705$$

Section 17

PERPETUITIES

Perpetuities

What Is a Perpetuity?

A perpetuity is a stream of equal cash flows that continues indefinitely.

Examples may include:

- Perpetual preferred shares
- Endowment payments
- Certain government securities
- Permanent annual scholarships

Present Value of a Perpetuity

$$PV = PMT / r$$

Where:

- PMT = constant periodic cash flow
- r = required return

Perpetuity Example

An investment pays ETB 50,000 every year indefinitely. Required return: 10%.

$$PV = 50{,}000 / 0.10$$

$$PV = \text{ETB } 500{,}000$$

Growing Perpetuity

A growing perpetuity increases at a constant rate forever.

$$PV = CF_1 / (r - g)$$

Where:

- CF_1 = next-period cash flow
- r = required return
- g = constant growth rate
- Condition: $r > g$

Growing-Perpetuity Example

Assume:

- Next cash flow: ETB 50,000
- Required return: 12%
- Growth rate: 4%

$$PV = 50{,}000 / (0.12 - 0.04)$$

$$PV = ETB\ 625{,}000$$

Section 18

UNEVEN CASH FLOWS

Uneven Cash Flows

Uneven Cash-Flow Stream

Uneven cash flows differ from period to period.

Examples include:

- Project revenues
- Investment returns
- Asset sale proceeds
- Seasonal cash flows
- Business expansion benefits

Present Value of Uneven Cash Flows

Each cash flow must be discounted separately.

$$PV = \sum CF_t \,/\, (1 + r)^t \qquad \text{for } t = 1 \text{ to } n$$

Uneven Cash-Flow Example

Required return: 10%.

Year	Cash Flow
1	ETB 100,000
2	ETB 150,000
3	ETB 200,000
4	ETB 250,000

Discount the Cash Flows

$$PV_1 = 100{,}000 / 1.10 = 90{,}909$$

$$PV_2 = 150{,}000 / 1.10^2 = 123{,}967$$

$$PV_3 = 200{,}000 / 1.10^3 = 150{,}263$$

$$PV_4 = 250{,}000 / 1.10^4 = 170{,}753$$

Total Present Value

$$PV = 90{,}909 + 123{,}967 + 150{,}263 + 170{,}753$$

$$PV \approx \text{ETB } 535{,}892$$

Future Value of Uneven Cash Flows

To find the future value at the end of Year n:

$$FV_n = \sum CF_t(1 + r)^{n-t} \quad \text{for } t = 0 \text{ to } n$$

Each cash flow compounds for a different number of periods.

DEFERRED & GROWING

Deferred and Growing Annuities

Deferred Annuity

A deferred annuity begins after a waiting period.

Examples:

- Retirement benefits beginning after 10 years
- Grace-period loan repayments
- Delayed lease payments
- Deferred scholarship payments

Valuing a Deferred Annuity

Steps:

1. Calculate the annuity value one period before the first payment
2. Discount that value back to today
3. Confirm the exact number of deferred periods

Growing Annuity

A growing annuity provides cash flows that increase at a constant rate for a limited number of periods.

$$PV = \left[CF_1 / (r - g) \right] \times \left[1 - ((1 + g)/(1 + r))^n \right]$$

Growing-Annuity Applications

Growing annuities may represent:

- Salaries
- Rent payments
- Revenue forecasts
- Maintenance costs
- Tuition expenses
- Dividend streams
- Pension contributions

Section 20

LOAN AMORTIZATION

Loan Amortization

What Is Loan Amortization?

Loan amortization is the gradual repayment of a loan through scheduled payments.

Each payment normally contains:

- Interest
- Principal repayment

Loan-Payment Components

Early in the Loan

- Interest portion is high
- Principal portion is low

As the Balance Declines

- Interest portion falls
- Principal portion increases

Loan-Amortization Process

For each period:

1. Calculate interest on opening balance
2. Subtract interest from total payment
3. Determine principal repayment
4. Reduce outstanding balance
5. Repeat for the next period

Amortization Formulas

$$\text{Interest} = \text{Opening Balance} \times r$$

$$\text{Principal Repaid} = \text{PMT} - \text{Interest}$$

$$\text{Closing Balance} = \text{Opening Balance} - \text{Principal Repaid}$$

Amortization Example

Assume:

- Loan: ETB 500,000
- Annual interest: 12%
- Term: 5 years
- Annual payment: ETB 138,705

$$\text{Year 1 Interest} = 500{,}000 \times 12\% = \text{ETB } 60{,}000$$

$$\text{Year 1 Principal} = 138{,}705 - 60{,}000 = \text{ETB } 78{,}705$$

Year 1 Closing Balance

500,000 − 78,705 = ETB 421,295

Closing Balance = ETB 421,295

The Year 2 interest will be calculated on ETB 421,295.

Partial Amortization Schedule

Year	Opening	Payment	Interest	Principal	Closing
1	500,000	138,705	60,000	78,705	421,295
2	421,295	138,705	50,555	88,150	333,145
3	333,145	138,705	39,977	98,728	234,417

Grace Period

A loan grace period may apply to principal only, or to both principal and interest.

During a principal grace period:

- Interest is still paid
- Principal remains outstanding

When interest is capitalized, the loan balance may increase.

Balloon Payment

A Balloon Loan Includes

- Smaller periodic payments
- Large final payment

Balloon Structures Increase

- Refinancing risk
- Final-payment risk
- Liquidity pressure at maturity

Section 21

INFLATION

Inflation and Real Value

Nominal and Real Returns

Nominal Return

Includes inflation.

Real Return

Measures the increase in purchasing power after inflation.

Fisher Relationship

$$1 + r_n = (1 + r_r)(1 + i)$$

Where:

- r_n = nominal return
- r_r = real return
- i = inflation rate

Approximate Real Return

$$\text{Real Return} \approx \text{Nominal Return} - \text{Inflation}$$

This approximation is most suitable when rates are relatively low.

Exact Real-Return Example

Assume:

- Nominal return: 15%
- Inflation: 8%

$$r_r = (1.15 / 1.08) - 1$$

$$r_r \approx 6.48\%$$

Inflation Consistency

Use:

- Nominal cash flows with nominal discount rates
- Real cash flows with real discount rates

Do not discount inflation-adjusted cash flows using a real rate unless both are consistently defined.

Section 22

APPLICATIONS

Financial Decision Applications

TVM and Capital Budgeting

Capital budgeting uses TVM to calculate:

- Net Present Value
- Internal Rate of Return
- Discounted Payback Period
- Present value of project benefits
- Present value of investment costs

TVM and Bond Valuation

Bond value equals:

- Present value of coupon payments
- Plus present value of principal repayment

Changes in discount rates affect bond prices.

TVM and Stock Valuation

Stock valuation uses TVM to discount:

- Expected dividends
- Free cash flows
- Terminal values
- Future sale prices

TVM and Retirement Planning

Retirement planning estimates:

- Required future retirement fund
- Periodic savings contribution
- Investment growth
- Retirement-period withdrawals
- Effect of inflation
- Required rate of return

TVM and Loan Decisions

TVM helps borrowers compare:

- Interest rates
- Monthly payments
- Loan terms
- Total interest
- Repayment structures
- Effective annual cost
- Refinancing alternatives

TVM and Lease Decisions

Lease analysis compares:

- Present value of lease payments
- Purchase cost
- Tax effects
- Maintenance responsibilities
- Residual value
- Financing alternatives

Section 23

SPREADSHEETS

Spreadsheet Applications

Excel TVM Functions

Common spreadsheet functions include:

- FV
- PV
- PMT
- RATE
- NPER
- NPV
- IRR
- XNPV
- XIRR

Excel FV Function

Syntax and example:

```
=FV(rate, nper, pmt, pv, type)
```

```
=FV(10%, 5, 0, -50000, 0)
```

This calculates the future value of ETB 50,000 invested for five years at 10%.

Excel PV Function

Syntax and example:

```
=PV(rate, nper, pmt, fv, type)
```

```
=PV(10%, 3, 0, -100000, 0)
```

This calculates the present value of ETB 100,000 received after three years.

Excel PMT Function

Syntax and example:

```
=PMT(rate, nper, pv, fv, type)
```

```
=PMT(12%, 5, -500000, 0, 0)
```

This calculates the annual payment on a five-year ETB 500,000 loan at 12%.

Spreadsheet Sign Convention

Spreadsheet financial functions distinguish cash inflows from cash outflows.

- Cash inflows
- Cash outflows

If the initial investment is entered as negative, the future value or payment generally appears as positive. Consistent signs are essential.

COMMON MISTAKES

Common TVM Mistakes

Common Calculation Errors

- Using the wrong rate per period
- Using the wrong number of periods
- Mixing annual and monthly rates
- Confusing ordinary annuity and annuity due
- Using future value instead of present value
- Ignoring compounding frequency
- Adding cash flows from different periods
- Ignoring inflation
- Using nominal rates with real cash flows
- Misapplying spreadsheet signs

Rate-and-Period Consistency

Cash-Flow Frequency	Rate	Number of Periods
Annual	Annual rate	Years
Semiannual	Annual rate ÷ 2	Years × 2
Quarterly	Annual rate ÷ 4	Years × 4
Monthly	Annual rate ÷ 12	Years × 12

TVM Problem-Solving Steps

1. Identify the question
2. Draw a timeline
3. Identify cash flows
4. Determine payment timing
5. Match rates and periods
6. Select the formula
7. Calculate
8. Interpret the result
9. Test reasonableness

PRACTICAL EXERCISE

Practical Exercise

Apply Time Value of Money Techniques

Participants will calculate:

- Future value
- Present value
- Effective annual rate
- Annuity value
- Required savings contribution
- Loan payment
- Uneven cash-flow value
- Real rate of return

Exercise 1 — Future Value

An organization invests ETB 500,000 for five years at 12% compounded annually. Calculate the future value.

$$FV = 500{,}000\ (1.12)^5$$

Exercise 1 — Solution

$$FV = 500{,}000 \times 1.76234$$

$$FV \approx ETB\ 881{,}171$$

Exercise 2 — Present Value

A project expects to receive ETB 2,000,000 after four years. Required return: 14%. Calculate the present value.

$$PV = 2{,}000{,}000 \,/\, (1.14)^4$$

Exercise 2 — Solution

PV ≈ ETB 1,184,160

The future ETB 2 million is worth approximately ETB 1.184 million today at a 14% required return.

Exercise 3 — Effective Annual Rate

A bank offers a nominal rate of 18% compounded monthly. Calculate the effective annual rate.

$$EAR = (1 + 0.18/12)^{12} - 1$$

Exercise 3 — Solution

$$EAR = (1.015)^{12} - 1$$

$$EAR \approx 19.56\%$$

Exercise 4 — Savings Annuity

An individual deposits ETB 50,000 at each year-end for eight years at 10%. Calculate the accumulated amount.

$$FV = 50{,}000 \times \left[\left((1.10)^8 - 1\right) / 0.10 \right]$$

Exercise 4 — Solution

$$FV \approx ETB\ 571{,}795$$

Exercise 5 — Loan Payment

A business borrows ETB 2,000,000 at 15% for five years, with equal annual repayments. Calculate the annual payment.

$$PMT = (2{,}000{,}000 \times 0.15) / (1 - (1.15)^{-5})$$

Exercise 5 — Solution

PMT ≈ ETB 596,631

Each annual payment includes both interest and principal.

Exercise 6 — Uneven Cash Flows

A project generates the following cash flows. Required return: 12%. Calculate the total present value.

Year	Cash Flow
1	ETB 300,000
2	ETB 400,000
3	ETB 600,000
4	ETB 700,000

Exercise 6 — Solution

$$PV = 300{,}000/1.12 + 400{,}000/1.12^2 + 600{,}000/1.12^3 + 700{,}000/1.12^4$$

PV ≈ ETB 1,457,535

Exercise 7 — Real Return

An investment earns 20% while inflation is 12%. Calculate the exact real return.

$$r_r = (1.20 / 1.12) - 1$$

$$r_r \approx 7.14\%$$

Group Assignment

Each group should solve and present:

- One future-value problem
- One present-value problem
- One annuity problem
- One loan-payment problem
- One effective-rate problem
- One uneven cash-flow problem
- One decision recommendation

CHAPTER REVIEW

Chapter Review

Knowledge Check

- Why is money today worth more than money in the future?
- What is the difference between simple and compound interest?
- What is future value?
- What is present value?
- How does compounding frequency affect value?
- What is an effective annual rate?
- How does an annuity due differ from an ordinary annuity?
- What is a perpetuity?
- How are uneven cash flows valued?
- Why must rates and periods be consistent?

Knowledge Check Answers

- It can earn a return and provides immediate purchasing power
- Compound interest earns interest on previous interest
- The amount a current sum will grow to
- Today's equivalent of a future cash flow
- More frequent compounding increases effective return
- The actual annual rate after compounding
- An annuity due pays at the beginning of each period
- An equal cash-flow stream continuing indefinitely
- Each cash flow is discounted separately
- Mismatched rates and periods produce incorrect results

Action Plan

Identify one financial decision in your organization and calculate:

- Current cash flow
- Future cash flow
- Timing
- Interest or discount rate
- Present value
- Future value
- Payment frequency
- Effective annual cost or return
- Financial implication
- Recommended decision

Chapter Summary

- Money has value based on when it is received or paid
- Compounding moves cash flows forward in time
- Discounting moves cash flows backward in time
- Compound interest includes interest on accumulated interest
- Present value falls as time and discount rates increase
- Future value increases as time and interest rates increase
- Effective annual rate reflects compounding frequency
- Annuities represent equal periodic cash flows
- Perpetuities continue indefinitely
- Uneven cash flows must be valued individually
- Loan amortization separates payments into interest and principal
- TVM is fundamental to investment, financing and valuation decisions

Key Message

> ***Financial amounts cannot be compared meaningfully unless their timing is considered.***

Sound financial decisions require managers to connect:

Cash Flow › Time › Interest Rate › Risk › Present Value › Future Value › Decision

Questions and Discussion

THANK YOU

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Understanding the time value of money enables managers and investors to compare financial choices across time — transforming future promises and financing obligations into values that can be evaluated consistently for sound decisions.