COURSE: INVESTMENTS |CHAPTER 6

EFFICIENT DIVERSIFICATION

The Eggs-in-One-Basket Idea

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Chapter Purpose

This chapter explains one of the most important ideas in modern investment management.

Don't evaluate investments only one at a time.

Instead, ask:

What happens when we combine them into a portfolio?

And learn:

How diversification can reduce portfolio risk.

The shift is from single-asset thinking to portfolio thinking.

Learning Objectives

By the end of this chapter, students should be able to:

- Calculate portfolio expected return.
- Explain portfolio variance and standard deviation.
- Explain covariance and correlation.
- Demonstrate diversification mathematically.
- Distinguish systematic and firm-specific risk.
- Analyze two-asset portfolios.

- Explain the minimum-variance portfolio.
- Interpret the efficient frontier.
- Explain the role of a risk-free asset.
- Interpret the Capital Allocation Line.
- Explain the optimal risky portfolio.
- Connect risk aversion with portfolio choice.

Start With a Simple Question

Suppose you have ETB 1,000,000 to invest. Would you put it into...

One company

The entire amount concentrated in a single investment.

Several different investments

The amount spread across many holdings.

Most investors prefer the second approach. Why? Diversification.

Concentration magnifies the consequences of being wrong about any one investment.

The Eggs-in-One-Basket Idea

Imagine carrying 20 eggs.

Option A — one basket

If the basket falls, all 20 may break.

Option B — several baskets

If one falls, only part of your eggs are affected.

This is the intuition behind diversification.

But Diversification Is More Sophisticated

Simply owning many investments is not enough. Suppose you own shares in:

Five airlines

Five hotels

Five travel companies

A major collapse in tourism could affect all of them.

Therefore: effective diversification depends on how investments behave relative to one another.

What Is a Portfolio?

A portfolio is a collection of investments owned by an investor. It may contain:

- **Stocks**
- **Bonds**
- **Cash**
- **Real estate**
- **Investment funds**
- **Other assets**

Portfolio theory analyses the whole collection, not each holding in isolation.

Portfolio Example

An investor allocates ETB 1,000,000 as follows:

Investment	Amount (ETB)	Weight
Stocks	500,000	50%
Bonds	300,000	30%
Cash	200,000	20%
Total	**1,000,000**	**100%**

These percentages are called portfolio weights.

Portfolio Weight

The weight of an investment is its share of total portfolio value:

$$wi = Amount\ invested\ in\ asset\ i \div Total\ portfolio\ value$$

$$\Sigma\, wi = 1 \quad (that\ is,\ 100\%)$$

Portfolio weights normally sum to one; they describe how the money is distributed.

Portfolio Expected Return

Portfolio expected return is simply the weighted average of the expected returns of the investments.

$$E(Rp) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

Weights

How much of the portfolio each asset represents.

Expected returns

What each asset is expected to earn.

Result

A single weighted-average portfolio return.

Simple Example

Stock A

Weight = 60% · Expected return = 15%

Bond B

Weight = 40% · Expected return = 7%

$$E(Rp) = (0.60)(15\%) + (0.40)(7\%)$$
$$E(Rp) = 11.8\%$$

Expected portfolio return = 11.8%

Interpretation

The portfolio consists of 60% Stock + 40% Bond and has an expected return of 11.8%.

7%
Bond expected return

11.8%
Portfolio expected return

15%
Stock expected return

11.8% lies between 7% and 15% because portfolio return is a weighted average.

Is Portfolio Risk Also a Weighted Average?

No.

This is one of the most important lessons in portfolio theory.

Portfolio return

IS a weighted average of individual expected returns.

Portfolio risk

Depends on individual asset risks AND on how the assets move together.

Why Does Co-Movement Matter?

Asset A

Performs well when the economy grows.

Asset B

Performs relatively well when economic conditions weaken.

Their movements may partially offset each other.

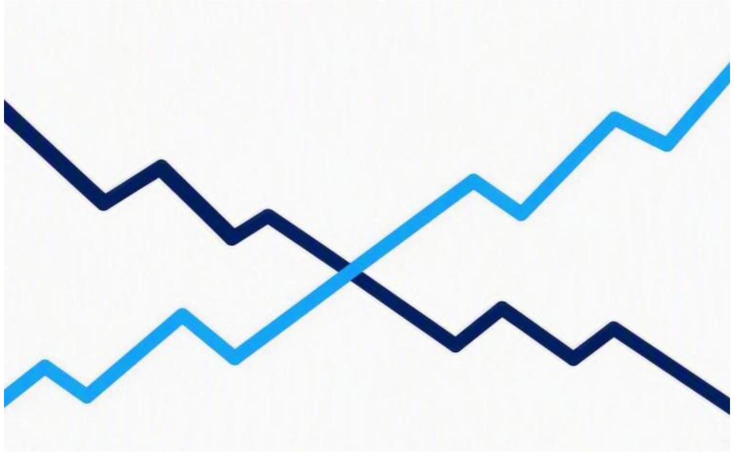
Result: portfolio volatility can be lower than the volatility of the parts.

The Diversification Effect

Consider two risky investments. If they do not move exactly together:

Losses in A
may be partly offset by

Gains or smaller losses in B
reducing overall portfolio variability.



Remember: Standard Deviation

Standard deviation measures the dispersion of possible returns around the expected return.

Larger σ

Greater volatility — outcomes are spread widely around the mean.

Smaller σ

Lower volatility — outcomes cluster near the mean.

Volatility is the standard measure of investment risk in mean-variance analysis.

Portfolio Standard Deviation

Portfolio standard deviation measures the volatility of the entire portfolio. It depends on:

1. **Asset weights**
2. **Individual volatility**
3. **Relationships among returns**

The third factor is what creates the diversification benefit.

What Is Covariance?

Covariance tells us whether two assets tend to move together or in opposite directions.

Together

Returns rise and fall at the same time.

In opposite directions

One rises while the other falls.

Think of it as answering: when Asset A changes, what does Asset B tend to do?

Positive Covariance

Asset A ↑ → Asset B ↑

Both rise together.

Asset A ↓ → Asset B ↓

Both fall together.

The two assets tend to move in the same direction.

Covariance > 0

Negative Covariance

Asset A ↑ → Asset B ↓

...or vice versa.

The two assets tend to move in opposite directions.

Covariance < 0

This can create powerful diversification benefits.

Covariance Near Zero

If the movements of two assets have little linear relationship:

$$\text{Covariance} \approx 0$$

Little shared information

Knowing the movement of one asset provides relatively little information about the movement of the other.

What Is Correlation?

Correlation standardizes covariance into an easier scale, independent of units.

$$\rho = Cov(A, B) \div (\sigma A\ \sigma B)$$

$$-1 \le \rho \le +1$$

Correlation always lies between −1 and +1, which makes assets easy to compare.

Understanding Correlation

+1

Assets move perfectly together.

0

No linear correlation.

−1

Assets move exactly opposite.

Diversification potential increases as correlation moves from +1 toward −1.

Correlation — Explained Simply

Imagine two people walking.

$\rho = +1$

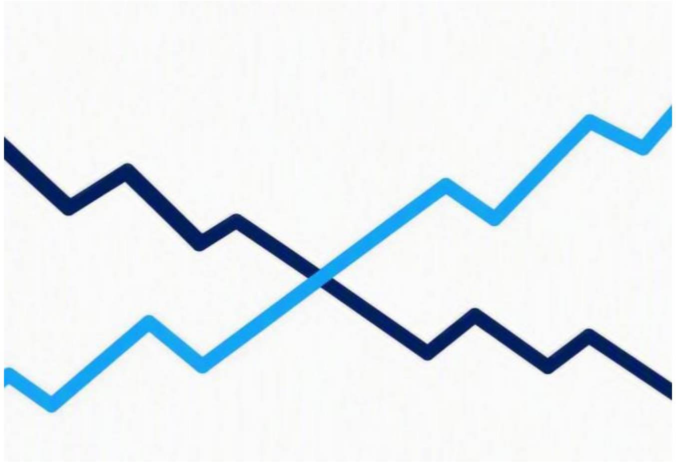
They always walk in the same direction.

$\rho \approx 0$

Their movements are largely unrelated.

$\rho = -1$

Whenever one walks left, the other walks right.



Which Is Better for Diversification?

Portfolio A

Assets with correlation $\rho = +1$

Portfolio B

Assets with correlation $\rho = 0.2$

All else equal, Portfolio B offers greater diversification potential.

Lower correlation leaves more scope for offsetting movements between the assets.

Key Diversification Principle

As correlation between portfolio assets decreases:

Potential Diversification Benefit ↑

Therefore

Investors should not only search for good investments. They should search for investments that work well together.

Two-Asset Portfolio

Suppose the portfolio contains Asset A and Asset B. Portfolio risk depends on:

wA
Weight of A

wB
Weight of B

σA
Risk of A

σB
Risk of B

...and the correlation between A and B

Two-Asset Portfolio Variance

$$\sigma p^2 = wA^2\sigma A^2 + wB^2\sigma B^2 + 2\, wA\, wB\, \sigma A\, \sigma B\, \rho AB$$

First two terms

The individual risk contributions of A and B.

Final term

Captures how the two assets move together.

The covariance term is where diversification enters the mathematics.

Why Correlation Matters

Look at the final part of the variance formula:

$$2\, wA\, wB\, \sigma A\, \sigma B\, \rho AB$$

If correlation decreases → portfolio variance decreases

...assuming everything else remains unchanged. That is the mathematics behind diversification.

Numerical Example

Asset A

E(R) = 14% · σ = 20%

Asset B

E(R) = 8% · σ = 10%

Portfolio

50% A + 50% B

Correlation

ρ = 0.20

We now compute the portfolio's expected return, variance and standard deviation.

Step 1 — Expected Portfolio Return

$$E(Rp) = (0.50)(14\%) + (0.50)(8\%)$$
$$E(Rp) = 11\%$$

Expected Portfolio Return = 11%

As always, the portfolio's expected return is the weighted average of the two returns.

Step 2 — Portfolio Variance

Using decimals:

$$\sigma p^2 = (0.5)^2(0.20)^2 + (0.5)^2(0.10)^2$$
$$+\,2(0.5)(0.5)(0.20)(0.10)(0.20)$$
$$\sigma p^2 = 0.0145$$

Each term is expressed in decimals so the result is a variance, not a percentage.

Step 3 — Portfolio Standard Deviation

$$\sigma p = \sqrt{0.0145} \approx 12.04\%$$

11%

Expected return

12.04%

Portfolio risk (σ)

What Did Diversification Do?

20%
Asset A risk

10%
Asset B risk

15%
Simple 50/50 average

12.04%
Actual portfolio risk

Why? The assets are not perfectly positively correlated.

The gap between 15% and 12.04% is the diversification effect, measured.

What If Correlation = +1?

$\rho = +1$ — the two assets move perfectly together

No offsetting movement

There is no risk reduction from imperfect co-movement.

Weakest case

Portfolio risk is simply the weighted average of the two standard deviations.

Diversification benefits are therefore at their weakest.

What If Correlation = −1?

$\rho = -1$ — assets move perfectly in opposite directions

With the appropriate weights

It may theoretically be possible to construct a portfolio with zero portfolio variance.

The strongest theoretical diversification case

Not All Risk Is the Same

Investment risk can be divided conceptually into two components:

Firm-Specific Risk

Affects a particular company or narrow group of companies.

Systematic Risk

Affects many securities across the whole market.

This distinction is fundamental to portfolio theory — and to the next chapter.

Firm-Specific Risk

Firm-specific risk arises from factors affecting a particular company or a relatively narrow group of companies.

EXAMPLES

- Management failure
- Product failure
- Employee strike
- Factory fire
- Lawsuit
- Loss of a major customer

Can Firm-Specific Risk Be Diversified?

Yes — substantially.

Example

Suppose one company experiences a product failure. Other companies in a diversified portfolio may remain unaffected.

Diversifiable risk

Alternative name

Idiosyncratic risk

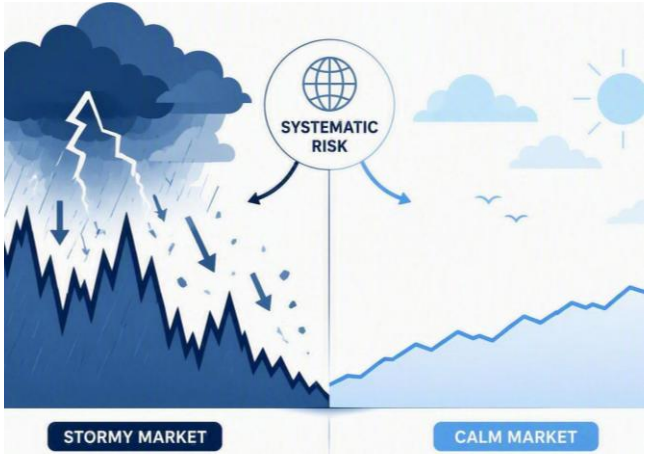
Alternative name

Systematic Risk

Systematic risk arises from broad economic or market factors affecting many securities.

EXAMPLES

- Recession
- Interest-rate changes
- Inflation shocks
- Financial crises
- Broad geopolitical shocks



Can Systematic Risk Be Diversified Away?

Not simply by adding more securities from the same broad market.

Because broad market forces affect many investments simultaneously.

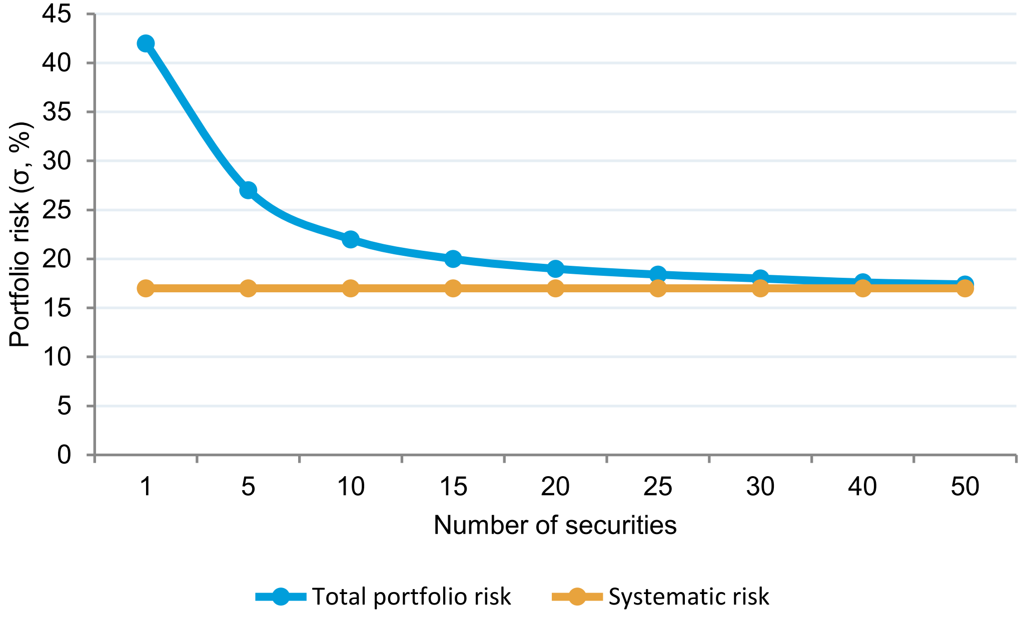
Market Risk

Alternative name

Non-diversifiable Risk

Alternative name

Diversification and Number of Securities



Firm-Specific Risk ↓

Falls as securities are added

Systematic Risk

Remains

Total Risk = Systematic + Firm-Specific

The Key Lesson

Investors should not expect additional return merely for bearing avoidable firm-specific risk.

Why?

Because investors can reduce much of that risk simply through diversification.

This idea becomes extremely important in the CAPM, in the next chapter.

What Is a Portfolio Opportunity Set?

Suppose we change the proportions invested in Assets A and B. For example:

100% A / 0% B	**80% A / 20% B**	**50% A / 50% B**	**20% A / 80% B**	**0% A / 100% B**

Each combination produces a different expected return–risk combination

Plotting the Portfolio Opportunity Set

Plot every possible portfolio of the two assets:

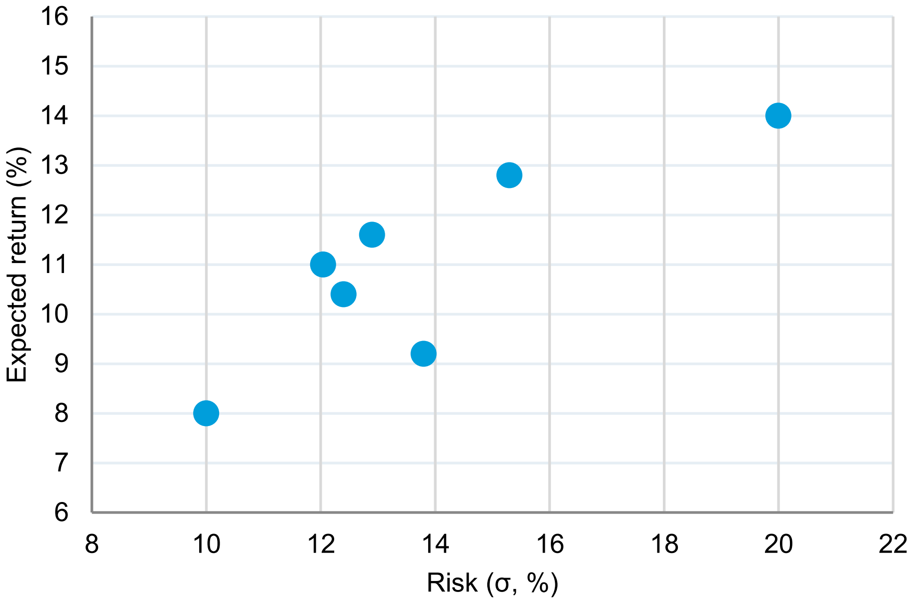
Horizontal axis

Portfolio risk (σ)

Vertical axis

Expected return E(R)

The resulting combinations form a portfolio opportunity set.



What Is the Minimum-Variance Portfolio?

Among all possible combinations of risky assets, one portfolio has the:

Lowest Possible Variance

Global Minimum-Variance Portfolio

The left-most point of the portfolio opportunity set — no other combination of these risky assets is less volatile.

Minimum Variance — Explained Simply

Imagine several routes to your destination. You ask:

"Which route gives me the least bumpy ride?"

The answer

The minimum-variance portfolio is the portfolio with the lowest volatility available from the considered risky assets.

What Is an Efficient Portfolio?

An efficient portfolio provides:

Highest expected return
for a given level of risk

Lowest risk
for a given expected return

The two statements are equivalent ways of saying the same thing.

Inefficient Portfolio

Portfolio A

Expected return = 10% · Risk = 12%

Portfolio B

Expected return = 10% · Risk = 18%

Portfolio A is preferable — same expected return, less risk.

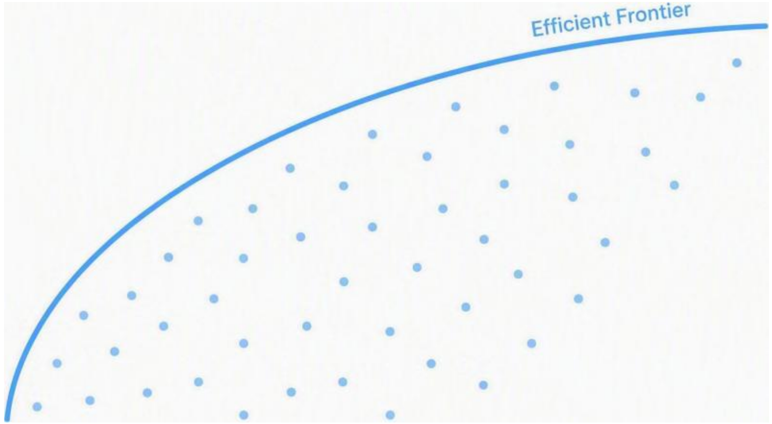
Portfolio B is therefore inefficient relative to A.

What Is the Efficient Frontier?

The efficient frontier is the set of efficient risky portfolios offering the best available expected-return / risk combinations.

Upper efficient portion

It forms the upper part of the portfolio opportunity set — everything below it is dominated.



Efficient Frontier — Explained Simply

Imagine shopping for a laptop. For every price level, you want the best performance available — you would reject a laptop if another costs the same but performs better.

Similarly, investors reject a portfolio when another portfolio offers:

More expected return
for the same risk

Less risk
for the same expected return

Efficient versus Inefficient Portfolios

On the efficient frontier

Best available risk–return combinations.

Below the efficient frontier

Inferior combinations — dominated by others.

Rational mean-variance investors focus on portfolios located on the efficient frontier.

What Is a Risk-Free Asset?

In portfolio theory, a risk-free asset is an idealized investment whose return over the relevant horizon is known with certainty.

$$\sigma f = 0$$

In practice

Short-term government securities are often used as practical proxies, subject to assumptions.

Combining Risk-Free and Risky Assets

Risk-Free Asset

Certain return, zero volatility

Risky Portfolio P

Higher expected return, positive σ

$$E(RC) = (1 - y)\, Rf + y\, E(RP)$$

where y is the proportion invested in the risky portfolio.

What Is the Capital Allocation Line?

The Capital Allocation Line (CAL) shows the possible expected-return / risk combinations obtained by mixing:

A Risk-Free Asset

A Particular Risky Portfolio

Every point on the line is one allocation between the two.

The Slope of the CAL

The slope of the Capital Allocation Line is:

$$[\,E(RP) - Rf\,] \div \sigma P$$

It measures expected excess return per unit of portfolio risk

A steeper CAL means more expected reward for each unit of volatility accepted.

CAL Example

$R_f = 4\%$
Risk-free rate

$E(R_P) = 12\%$
Risky portfolio return

$\sigma_P = 16\%$
Risky portfolio risk

$$(12\% - 4\%) \div 16\% = 0.50$$

The portfolio offers 0.50 units of expected risk premium per unit of volatility.

What Is the Optimal Risky Portfolio?

When a risk-free asset is available, investors seek the risky portfolio that provides the most attractive expected excess return relative to volatility.

Graphically

It is associated with the CAL that is tangent to the efficient frontier under the standard framework.

The steepest attainable CAL identifies the best risky portfolio to hold.

The Tangency Portfolio

The point where the best attainable CAL touches the efficient frontier is called:

The Tangency Portfolio

It is the optimal risky portfolio

...under the standard mean-variance framework with common assumptions.

What Is Risk Aversion?

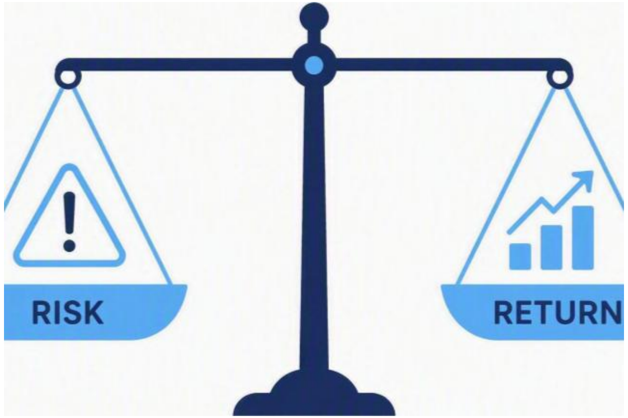
Risk aversion means investors prefer less risk when expected returns are otherwise equal.

Portfolio A

Return = 10% · Risk = 8%

Portfolio B

Return = 10% · Risk = 15%



A risk-averse investor prefers Portfolio A.

Investors Have Different Risk Tolerances

Conservative

May allocate more to the risk-free asset.

Moderate

May combine substantial amounts of both.

Aggressive

May allocate more to the risky portfolio.

Investors can hold the same optimal risky portfolio, yet choose different allocations between risky and risk-free assets.

Practical Exercise

Investor A
Highly risk-averse · ETB 1,000,000

Investor B
Moderate tolerance · ETB 1,000,000

Investor C
High tolerance · ETB 1,000,000

Risk-free asset: 5%
Available to all investors

Risky portfolio: E(R) = 15%, σ = 20%
Available to all investors

STUDENT TASK Recommend how each investor might divide ETB 1,000,000 between the risk-free asset and the risky portfolio — then explain why the allocations should differ.

Key Takeaways

- Portfolio return is a weighted average of returns.
- Portfolio risk is not a weighted average of risks.
- Covariance shows how asset returns move together.
- Correlation standardizes that relationship: −1 to +1.
- Lower correlation means greater diversification potential.
- Diversification substantially reduces firm-specific risk.
- Systematic risk cannot be diversified away.

- The minimum-variance portfolio has the lowest variance.
- Efficient portfolios give the best risk–return mix.
- The efficient frontier holds all efficient portfolios.
- A risk-free asset combines with risky portfolios.
- The Capital Allocation Line maps those combinations.
- The tangency portfolio is the optimal risky portfolio.
- Risk aversion sets the final allocation.

REMEMBER Good diversification is not about owning many investments. It is about owning investments whose risks interact effectively.

Next Chapter — Chapter 7

Capital Asset Pricing and Arbitrage Pricing Theory. The next chapter will cover:

- Capital Asset Pricing Model
- Systematic risk and Beta
- Market portfolio
- Security Market Line
- Expected and required return

- Market risk premium and Alpha
- Undervalued / overvalued securities
- CAPM assumptions and limitations
- Multifactor models and APT
- Factor sensitivities · CAPM vs APT

Plus practical investment applications.

THANK YOU

Questions and Discussion

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