COURSE: INVESTMENTS |CHAPTER 16

OPTION VALUATION

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Chapter Purpose

This chapter explains:

- Intrinsic value
- Time value
- Option-price boundaries
- Factors affecting option values
- Put–call parity
- Synthetic positions
- One-period binomial valuation
- Multiperiod binomial valuation
- Risk-neutral probabilities
- Black–Scholes valuation
- Option Greeks
- Implied volatility
- Model limitations

Learning Objectives

By the end of this chapter, students should be able to:

1. Explain the components of an option premium.
2. Calculate call and put intrinsic value.
3. Calculate option time value.
4. Explain option-price boundaries.
5. Identify determinants of option value.
6. Apply put–call parity.
7. Explain synthetic positions.
8. Apply the one-period binomial model.
9. Calculate a hedge ratio.
10. Apply risk-neutral valuation.
11. Explain multiperiod binomial valuation.
12. Explain the Black–Scholes model.
13. Interpret option Greeks.
14. Explain implied volatility.
15. Identify model assumptions and limitations.

SECTION I

OPTION-VALUE FOUNDATIONS

What makes an option valuable?

What Is Option Value?

An option's market price is called its premium.

Premium

The premium reflects:

1. The value of exercising today
2. The possibility of becoming more valuable before expiration

```
Option Premium = Intrinsic Value + Time Value
```

Two Sources of Value

Imagine a coupon that lets you buy a laptop for ETB 100,000.

Value Today

If the laptop already costs ETB 120,000, the coupon immediately saves ETB 20,000.

Possible Future Value

Even if the laptop currently costs ETB 95,000, its price might rise before the coupon expires.

These represent intrinsic value and time value.

Call Intrinsic Value

The intrinsic value of a call is:

```
Call Intrinsic Value = max(S − K, 0)
```

Where:

- S = current underlying price
- K = strike price

A call has intrinsic value when $S > K$.

Call Intrinsic-Value Example

Suppose:

- Share price = ETB 120
- Call strike = ETB 100

```
Intrinsic Value = max(120 − 100, 0)
Intrinsic Value = ETB 20
```

Put Intrinsic Value

The intrinsic value of a put is:

$$\text{Put Intrinsic Value} = \max(K - S, 0)$$

A put has intrinsic value when the strike exceeds the share price:

$$K > S$$

Put Intrinsic-Value Example

Suppose:

- Put strike = ETB 100
- Share price = ETB 80

```
Intrinsic Value = max(100 − 80, 0)
Intrinsic Value = ETB 20
```

Time Value

```
Time Value = Option Premium – Intrinsic Value
```

Time value reflects the possibility of favourable price movement before expiration.

An out-of-the-money option may have:

- Zero intrinsic value
- Positive time value

Time-Value Example

Suppose:

- Share price = ETB 120
- Call strike = ETB 100
- Call premium = ETB 27

Intrinsic value:

```
120 – 100 = ETB 20
```

Time value:

```
27 – 20 = ETB 7
```

Value at Expiration

At expiration, no time remains for further price movement. Therefore time value = 0, and option value equals intrinsic value.

Call

```
C_T = max(S_T − K, 0)
```

Put

```
P_T = max(K − S_T, 0)
```

Call-Price Lower Bound

A European call on a non-dividend-paying share should generally satisfy:

$$C \geq \max(S - PV(K), 0)$$

Why?

The option should not sell for less than the immediate economic advantage implied by the share price and the present value of the strike.

Put-Price Lower Bound

A European put on a non-dividend-paying share should generally satisfy:

$$P \geq \max(PV(K) - S, 0)$$

The put's value cannot reasonably fall below this lower boundary without creating potential arbitrage.

Upper Bounds

Call Upper Bound

$$C \leq S$$

A call should not be worth more than the underlying share.

Put Upper Bound

$$P \leq PV(K)$$

For a European put, the most it can deliver at expiration is the strike price.

SECTION II

WHAT DRIVES OPTION VALUE?

Six major variables shape option premiums.

The Six Main Inputs

Option values are influenced by:

1. Underlying price
2. Strike price
3. Time to expiration
4. Volatility
5. Risk-free interest rate
6. Expected dividends

> *These six inputs flow into every option-pricing model.*

Underlying Price

Other things equal:

Share Price Increases

- Call value generally increases.
- Put value generally decreases.

Share Price Decreases

- Call value generally decreases.
- Put value generally increases.

Strike Price

Other things equal:

Higher Strike Price

- Reduces call value
- Increases put value

Lower Strike Price

- Increases call value
- Reduces put value

Why? The strike determines the contractual buying or selling price.

Time to Expiration

More time generally gives the underlying asset more opportunity to move favourably. Therefore, other things equal:

- More time → usually increases option value.
- Less time → usually reduces time value.

Important

The relationship may be more complicated for some dividend-paying or interest-sensitive positions.

Volatility

Volatility measures the uncertainty of the underlying asset's returns.

Higher volatility generally:

- Increases call value
- Increases put value

> *Greater uncertainty can benefit option holders because downside is limited while favourable movement may create value.*

Risk-Free Interest Rate

A higher risk-free interest rate generally:

- Increases European call value
- Decreases European put value

One reason is that a higher rate reduces the present value of the strike price paid or received in the future.

Expected Dividends

When a share pays a dividend, its price generally falls by approximately the dividend amount on the ex-dividend date, other things equal.

Therefore, expected dividends generally:

- Reduce call value
- Increase put value

Directional Summary

Factor Increases	Call Value	Put Value
Underlying price	Increases	Decreases
Strike price	Decreases	Increases
Time to expiration	Usually increases	Usually increases
Volatility	Increases	Increases
Risk-free rate	Increases	Decreases
Expected dividends	Decreases	Increases

SECTION III

PUT–CALL PARITY

Different portfolios with identical future payoffs should have identical values today.

The Law of One Price

If two investments produce exactly the same future cash flows under all possible outcomes, they should have the same current price.

Otherwise:

- Buy the cheaper portfolio.
- Sell the more expensive portfolio.
- Lock in a potential arbitrage profit.

This principle supports put–call parity.

Two Equivalent Portfolios

Consider European options with the same underlying share, strike price and expiration date.

Portfolio A

- Long call
- Risk-free investment equal to PV(K)

Portfolio B

- Long put
- Long share

At expiration, both portfolios produce the same payoff.

Put–Call Parity Formula

For European options on a non-dividend-paying share:

$$C + PV(K) = P + S$$

Where:

- C = call price
- P = put price
- S = share price
- PV(K) = present value of the strike price

— Keeping the Scales Balanced

Put–call parity is like a balanced scale.

Left Side

Call plus enough risk-free money to pay the strike.

Right Side

Put plus the underlying share.

> *Because both sides provide the same expiration payoff, they should have the same value today.*

Parity Example — Inputs

Suppose:

- Share price = ETB 100
- Strike price = ETB 100
- Time to expiration = One year
- Risk-free rate = 5%
- European call price = ETB 12
- No dividends

First calculate the present value of the strike.

Present Value of Strike

Using annual compounding:

$$PV(K) = 100 \div 1.05$$
$$PV(K) = ETB\ 95.24$$

Calculate the Put Price

From C + PV(K) = P + S, rearrange:

$$P = C + PV(K) - S$$

$$P = 12 + 95.24 - 100$$

$$P = \text{ETB } 7.24$$

Verify the Equality

Left Side

```
C + PV(K) = 12 + 95.24 = ETB 107.24
```

Right Side

```
P + S = 7.24 + 100 = ETB 107.24
```

The two portfolios are balanced.

Dividend-Adjusted Parity

If known dividends will be paid before expiration:

$$C + PV(K) + PV(\text{Dividends}) = P + S$$

Alternatively:

$$C - P = S - PV(\text{Dividends}) - PV(K)$$

Expected dividends generally reduce calls relative to puts.

Synthetic Share

Rearrange put–call parity:

$$S = C - P + PV(K)$$

Therefore, a synthetic long share can be constructed using:

- Long call
- Short put
- Risk-free investment equal to PV(K)

Synthetic Call

Rearrange:

$$C = P + S - PV(K)$$

A synthetic call consists of:

- Long put
- Long share
- Borrowing the present value of the strike

Synthetic Put

Rearrange:

$$P = C + PV(K) - S$$

A synthetic put consists of:

- Long call
- Risk-free investment
- Short share

Synthetic Risk-Free Investment

Rearrange:

$$PV(K) = P + S - C$$

This consists of:

- Long put
- Long share
- Short call

It produces the strike price at expiration.

Parity and Arbitrage

If observed option prices violate put–call parity, traders may attempt to:

1. Buy the cheaper portfolio.
2. Sell the more expensive portfolio.
3. Hold the positions until expiration.
4. Capture the price difference.

Practical Limitation

Transaction costs, bid–ask spreads, taxes, borrowing constraints and exercise rules may prevent a true arbitrage.

SECTION IV

BINOMIAL OPTION VALUATION

Build an option price from possible future share prices.

The Binomial Idea

The binomial model assumes that during one period, the underlying price can move to one of two values:

- Up state: S_u
- Down state: S_d

The option value is determined from its payoffs in these two states.

— Two Possible Roads

Suppose a share worth ETB 100 can take only two roads over the next year:

Up Road

Price becomes ETB 120.

Down Road

Price becomes ETB 80.

If we know the option's payoff at the end of each road, we can work backward to estimate its value today.

One-Period Example — Inputs

Suppose:

- Current share price = ETB 100
- Up-state price = ETB 120
- Down-state price = ETB 80
- Call strike = ETB 100
- Risk-free return for the period = 5%

> *These inputs define a one-period binomial tree.*

Terminal Share Prices

Up State

$$S_u = 100 \times 1.20 = \text{ETB } 120$$

Down State

$$S_d = 100 \times 0.80 = \text{ETB } 80$$

Terminal Call Payoffs

Call payoff: $C_T = \max(S_T - K, 0)$

Up-State Payoff

```
C_u = max(120 - 100, 0) = ETB 20
```

Down-State Payoff

```
C_d = max(80 - 100, 0) = 0
```

Replicating Portfolio

The option can be replicated using:

- Δ shares of the underlying
- A risk-free borrowing or investment

The hedge ratio is:

$$\Delta = (C_u - C_d) \div (S_u - S_d)$$

Calculate the Hedge Ratio

$$\Delta = (20 - 0) \div (120 - 80)$$

$$\Delta = 20 \div 40 = 0.50$$

A portfolio containing 0.5 share changes in value like one call across the two states.

Determine the Risk-Free Position

In the up state:

$$0.5(120) + 1.05B = 20$$
$$60 + 1.05B = 20$$
$$1.05B = -40$$
$$B = -\text{ETB } 38.10$$

The negative value represents borrowing.

Replication-Based Call Value

Today:

$$C_0 = \Delta \cdot S_0 + B$$

$$C_0 = 0.5(100) - 38.10$$

$$C_0 = \text{ETB } 11.90$$

The one-period binomial call value is approximately ETB 11.90.

Why Replication Works

The replicating portfolio produces exactly the same terminal payoff as the call.

State	0.5 Share	Loan Repayment	Net Payoff
Up	60	−40	20
Down	40	−40	0

> *By the law of one price, the option and the replicating portfolio must have the same value.*

SECTION V

RISK-NEUTRAL VALUATION

Use adjusted probabilities to discount option payoffs at the risk-free rate.

Risk-Neutral Probability

The risk-neutral up probability is:

$$q = (R - d) \div (u - d)$$

Where:

- R = one plus the risk-free return
- u = up factor
- d = down factor

Calculate the Probability

Given R = 1.05, u = 1.20 and d = 0.80:

$$q = (1.05 - 0.80) \div (1.20 - 0.80)$$
$$q = 0.25 \div 0.40 = 0.625$$

The risk-neutral down probability is $1 - q = 0.375$.

Risk-Neutral Call Value

$$C_0 = [q \cdot C_u + (1 - q) \cdot C_d] \div R$$

$$C_0 = [0.625(20) + 0.375(0)] \div 1.05$$

$$C_0 = 12.50 \div 1.05 = \text{ETB } 11.90$$

Risk-Neutral Put Value

Put payoffs:

- Up state: P_u = max(100 – 120, 0) = 0
- Down state: P_d = max(100 – 80, 0) = 20

Therefore:

$$P_0 = [0.625(0) + 0.375(20)] \div 1.05$$
$$P_0 = \text{ETB } 7.14$$

Put–Call Parity Check

$$C + PV(K) = P + S$$

Left side:

$$11.90 + (100 \div 1.05) = 107.14$$

Right side:

$$7.14 + 100 = 107.14$$

The binomial values satisfy put–call parity.

Risk-Neutral Is Not Real Probability

The risk-neutral probability is:

- A valuation tool
- Derived from no-arbitrage conditions
- Used to discount expected payoffs at the risk-free rate

It is not necessarily:

- The investor's actual forecast
- The historical probability
- The true probability of a price increase

No-Arbitrage Condition

For the binomial model to avoid immediate arbitrage:

$$d < R < u$$

In the example:

$$0.80 < 1.05 < 1.20$$

The risk-free return lies between the down and up returns.

SECTION VI

MULTIPERIOD BINOMIAL VALUATION

Divide the option's life into several short periods.

Two-Period Price Tree

Suppose S_0 = ETB 100, u = 1.20 and d = 0.80. After two periods:

$$S_{uu} = 100(1.20)^2 = 144$$

$$S_{ud} = 100(1.20)(0.80) = 96$$

$$S_{dd} = 100(0.80)^2 = 64$$

Terminal Call Payoffs

Assume K = ETB 100.

```
C_uu = max(144 – 100, 0) = 44
 C_ud = max(96 – 100, 0) = 0
 C_dd = max(64 – 100, 0) = 0
```

Work Back from the Up Node

Using q = 0.625:

```
C_u = [0.625(44) + 0.375(0)] ÷ 1.05
C_u = 27.50 ÷ 1.05
C_u = ETB 26.19
```

Work Back from the Down Node

Both terminal call payoffs following the first down movement are zero.

$$C_d = [0.625(0) + 0.375(0)] \div 1.05$$
$$C_d = 0$$

Calculate Today's Call Value

$$C_0 = [0.625(26.19) + 0.375(0)] \div 1.05$$

$$C_0 \approx \text{ETB } 15.59$$

This process is called:

Backward Induction

Why Add More Periods?

More periods allow the model to represent:

- More possible future prices
- More exercise opportunities
- Changing volatility
- Changing interest rates
- Dividend payments
- American-style exercise

As the number of periods increases, the binomial model can approximate continuous-time models.

American Option Valuation

For an American option, compare at every node:

Continuation Value

Value from keeping the option alive.

Exercise Value

Value from exercising immediately.

```
Option Value = Maximum(Continuation Value, Exercise Value)
```

Early Exercise of Calls

For a non-dividend-paying share, early exercise of an American call is generally not optimal. Reasons include:

- Exercise sacrifices remaining time value.
- Paying the strike early loses interest on the cash.
- Selling the option may be more valuable than exercising.

Dividends may change this conclusion.

Early Exercise of Puts

Early exercise of an American put may sometimes be attractive when:

- The put is deeply in the money.
- Little time value remains.
- Interest rates are positive.
- Immediate receipt of the strike is valuable.

The decision must compare exercise value with continuation value.

Binomial Model Strengths

- Intuitive
- Flexible
- Shows the evolution of value
- Handles American exercise
- Can incorporate dividends
- Can allow changing inputs
- Supports hedge-ratio calculations
- Works well for teaching

Binomial Model Limitations

- Requires assumptions about price movements.
- Results depend on volatility estimates.
- Large trees require more computation.
- Discrete steps simplify continuous markets.
- Transaction costs are often ignored.
- Hedge adjustments may not be continuous.
- Market prices may differ from model values.

SECTION VII

BLACK–SCHOLES OPTION VALUATION

A continuous-time model for European options.

What Is the Black–Scholes Model?

The Black–Scholes model estimates the value of European options using:

- Underlying price
- Strike price
- Time to expiration
- Risk-free interest rate
- Volatility
- Dividend yield, when applicable

It provides a closed-form option-pricing solution.

Basic Assumptions

The basic model assumes:

- European exercise
- No arbitrage
- Constant volatility
- Constant risk-free rate
- Continuous trading
- Frictionless markets
- No transaction costs
- Lognormally distributed share prices
- Continuous hedging
- Initially, no dividends

Black–Scholes Call Formula

For a non-dividend-paying share:

$$C_0 = S_0 \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

$$d_1 = [\ln(S_0/K) + (r + \sigma^2/2)T] \div (\sigma\sqrt{T})$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Understanding the Symbols

- S_0 = current share price
- K = strike price
- r = continuously compounded risk-free rate
- T = time to expiration
- σ = annual volatility
- $N(d)$ = cumulative standard normal probability
- e^{-rT} = discount factor

Black–Scholes Example — Inputs

Suppose:

- Share price = ETB 100
- Strike price = ETB 100
- Time to expiration = One year
- Risk-free rate = 5%
- Volatility = 20%
- No dividends

Calculate d_1

$$d_1 = [\ln(100/100) + (0.05 + 0.20^2/2)(1)] \div (0.20\sqrt{1})$$

$$d_1 = (0 + 0.07) \div 0.20$$

$$d_1 = 0.35$$

Calculate d_2

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_2 = 0.35 - 0.20 = 0.15$$

From the standard normal distribution:

$$N(d_1) \approx 0.6368$$

$$N(d_2) \approx 0.5596$$

Calculate the Call Price

$$C_0 = 100(0.6368) - 100 \cdot e^{(-0.05)}(0.5596)$$

$$C_0 \approx 63.68 - 53.23$$

$$C_0 \approx \text{ETB } 10.45$$

Calculate the Put Price

Using the Black–Scholes put formula:

$$P_0 = K \cdot e^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1)$$

The estimated value is:

$$P_0 \approx \text{ETB } 5.57$$

Verify Through Put–Call Parity

$$P = C + K \cdot e^{(-rT)} - S$$

$$P = 10.45 + 100 \cdot e^{(-0.05)} - 100$$

$$P = 10.45 + 95.12 - 100$$

$$P = \text{ETB } 5.57$$

The values are consistent.

Dividend-Yield Extension

For a share paying a continuous dividend yield q:

$$C_0 = S_0 \cdot e^{(-qT)} \cdot N(d_1) - K \cdot e^{(-rT)} \cdot N(d_2)$$

$$P_0 = K \cdot e^{(-rT)} \cdot N(-d_2) - S_0 \cdot e^{(-qT)} \cdot N(-d_1)$$

Expected dividends reduce the effective present value of the share component.

Black–Scholes Limitations

Actual markets may violate model assumptions because:

- Volatility changes.
- Interest rates change.
- Trading is not continuous.
- Transaction costs exist.
- Large price jumps occur.
- Liquidity varies.
- Returns may not follow the assumed distribution.
- American options allow early exercise.

> *A model value is an estimate — not a guaranteed market price.*

What Are the Greeks?

The Greeks measure how option value responds to changes in key inputs.

Greek	Sensitivity
Delta	Underlying price
Gamma	Change in delta
Theta	Passage of time
Vega	Volatility
Rho	Interest rate

Delta

Delta measures the approximate change in option value for a one-unit change in the underlying price.

$$\Delta = \partial(\text{Option Value}) \div \partial S$$

Call Delta in the Black–Scholes Example

$$\Delta_C = N(d_1) \approx 0.637$$

Interpreting Delta

A call delta of 0.637 suggests that, for a small price change:

If the Share Rises by ETB 1

```
Call price increases by about ETB 0.637
```

The approximation is local because delta changes as price and time change.

Put Delta

For a non-dividend-paying share:

$$\Delta\_P = N(d\_1) - 1$$

$$\Delta\_P = 0.6368 - 1 = -0.3632$$

A put generally loses value when the underlying share price rises.

Gamma

Gamma measures how much delta changes when the underlying price changes.

$$\Gamma = \partial\Delta \div \partial S$$

High gamma means:

- Delta changes rapidly.
- The position requires more frequent hedge adjustment.
- Option value is highly curved relative to the underlying price.

Theta, Vega and Rho

Theta

Measures sensitivity to the passage of time. Long options generally experience negative theta.

Vega

Measures sensitivity to volatility. Long calls and puts generally have positive vega.

Rho

Measures sensitivity to interest rates. Calls generally have positive rho; puts generally have negative rho.

Implied Volatility and Exercise

Implied Volatility

The volatility level that makes a pricing model equal the observed market option price. Unlike historical volatility, it is inferred from current option prices.

Student Exercise

Given S_0 = ETB 100, K = ETB 100, u = 1.20, d = 0.80, R = 1.05:

1. Calculate risk-neutral probability.
2. Calculate terminal call payoffs.
3. Calculate the call value.
4. Calculate the put value.
5. Verify put–call parity.
6. Explain why risk-neutral probability is not a forecast.

Answer check: q = 0.625 · C_0 = ETB 11.90 · P_0 = ETB 7.14

Key Takeaways and Discussion

Key Takeaways

- Option premium equals intrinsic value plus time value.
- Calls and puts have different intrinsic-value relationships.
- Volatility generally increases both call and put values.
- Time usually benefits option holders.
- Put–call parity links calls, puts, shares and risk-free investments.
- The binomial model values options through replication and backward induction.
- Risk-neutral probabilities are valuation tools, not forecasts.
- Black–Scholes provides a closed-form European-option value.
- Greeks measure option-price sensitivities.
- Implied volatility is extracted from market option prices.
- All models depend on assumptions and estimated inputs.

Discussion Questions

1. Why can an out-of-the-money option have value?
2. Why does volatility increase both call and put values?
3. What economic principle supports put–call parity?
4. Why is risk-neutral probability not a real-world forecast?
5. When is early exercise of an American option reasonable?
6. Why must delta hedges be adjusted?
7. What does implied volatility reveal?
8. Why might a market price differ from a model value?

NEXT CHAPTER

Chapter 17: Futures Markets

- Forward and futures contracts
- Long and short positions
- Contract specifications
- Futures payoffs
- Margin requirements
- Marking to market
- Hedging with futures
- Basis and basis risk
- Stock-index futures
- Interest-rate futures
- Currency futures
- Commodity futures
- Speculation and arbitrage
- Futures-market risks

THANK YOU

Questions and Discussion

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